Calculations

SAMPLING: RABBITS OR RADIOACTIVITY
Page 6

TEKTRONIX®

IN THIS STATISTICS ISSUE:

Foreground: 21 and 31 calculators.
Background: CALCULATIONS' Editor, Ted Hoff.

A SOURCE OF SIGNIFICANCE

Introducting CALCULATIONS, a new quarterly publication and an important part of Tektronix's manifold thrust into the world of computations. On August 2nd, 1973 Tektronix introduced two new calculators along with a system of peripherals. These programmable machines, known as the 21 and 31, were presented as an alternative to the high cost of calculations. To add to the appeal, Tektronix has also developed a system of software known as the Calculator Program libraries. These libraries offer efficiency in your math computations no matter what your field. Because the programs were developed especially with and for the 21 and 31, it's only natural that they're presented together. It's a hand and glove situation.

Okay, you say, I can understand the need for calculators and software, but why a magazine? We'll lay it out. Calculations are only a means to an end, as are the tools employed along the way. CALCULATIONS is one of those tools—but a very special one. We'll be dealing with the relationship between calculators and software. Each issue will be based on a particular software library, the background of that field, and its applications. Our new software programs have a lot to say about statistics, mathematics, engineering and more. As development proceeds, CALCULATIONS will have much to talk about.

Although our magazine is calculator oriented, it's not just a text book or operator's manual. It's more. CALCULATIONS is a two-way street, clearing house and sounding board. On the bill are people, application ideas, new products, books, and even a bit of history. We want to share these with you in the firm hope you'll share your experience and expertise with us. CALCULATIONS' goal is to be used and enjoyed. Instead of being stacked in a bottom drawer with assorted corporate reports and next year's vacation schedule, we want our efforts to see the light of day. Coffee cup stains and margin notations are often the signs of success. Our magazine is a nice combination, we think, of technology with the human touch.

Statistics flavor our first issue. We discuss its tremendous potential and its nuts and bolts while exhibiting a few of its historical highlights as a backdrop. Other nuggets range from the advantages of subroutines to a profile of our first 31 Calculator customer.

In the coming months, we'll be talking about math calculations in a wide range of disciplines, and that includes yours. No matter what size, shape or color your numbers come in, we want CALCULATIONS to be of help. The magazine should be fun to look at, informative to read, and hard to put down. We hope to be a center of math activity. With your questions, comments and concern . . . we can.

Lawrence L. Mayhew

Vice President & General Manager,
Information Display Division
Tektronix, Inc.

STATISTICS PROGRAM LIBRARY

There's always a choice. If you're maneuvering statistics through a TEKTRONIX 21 or 31, then you're doing a lot of calculations. If you're doing a lot of calculations, then you're pounding a lot of keys. If that's so, maybe you're spending more time with the questions than the answers. The choice? A lot of repetitive manual statistics programming or the TEKTRONIX Statistics Program Libraries. These libraries—*groups of software programs*—were developed by and for people using the new 21 and 31 Calculators. What's in it for you? If you're running statistics of any complexity through one of these machines, these libraries can offer you some distinct operation advantages. What kind of advantages? Increased versatility and efficiency for starters.

The library takes the form of documentation manuals, one each for the 21 and 31.

Let's say you're using a 21. The first section of the 21 manual deals with standard tests and distributions with the use of the 128 step memory. If you're using a machine with the optional 256 step memory pack, you can wade in a lot deeper. The second section of the manual spells out programs such as:

General statistics: random numbers, means, standard deviations.
Tests: T, F, χ^2 and contingency tables.
Distributions: binomial, normal χ^2, T and F.
Curve fitting: line, exponential, power, parabola, and three variables.
Analysis of variance: one-way and Yates.

So much for versatility . . . what about efficiency? That comes in the shape of "linking". What this means is that an analysis may be continued in a second program based on the results stored in the memory registers by the first program. For example, the

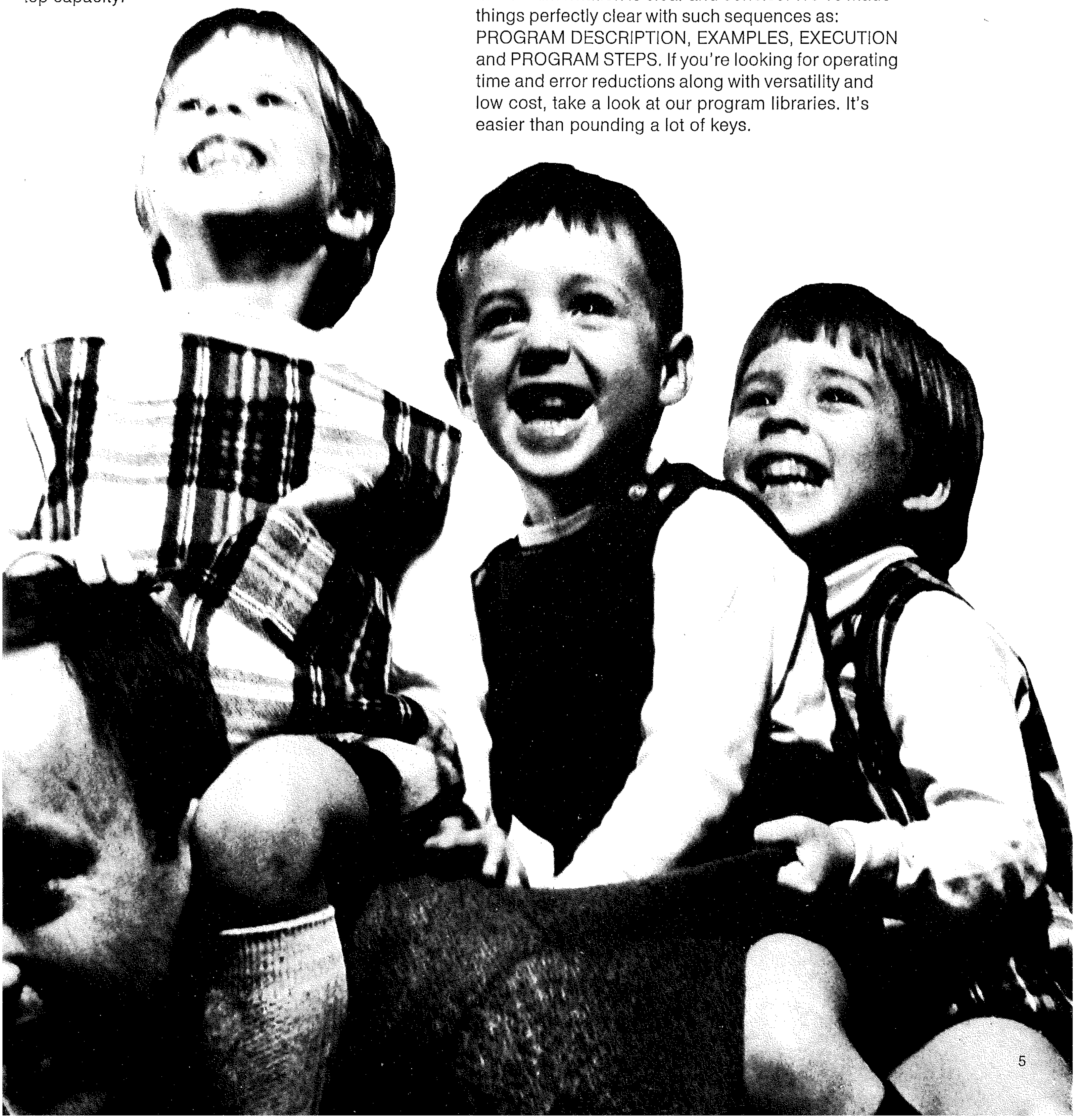
CHANCES ARE...

rograms in one section—*that calculate the T-statistic* –can be linked to the T-distribution programs in nother section. The T-statistic and degrees of 'eedom do not have to be re-entered.

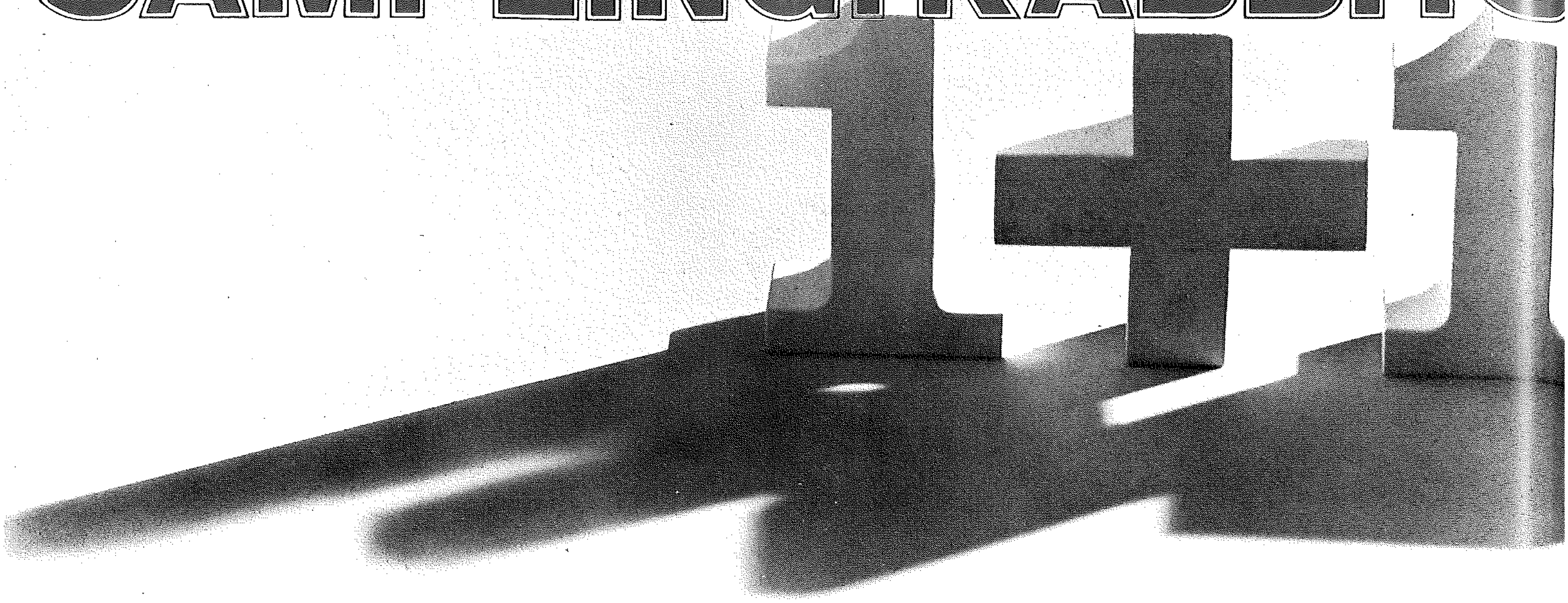
But maybe there's a 31 sitting on your desk. so, you've got a standard 512 step memory at your ngertips, and—*if you need them*—options are vailable that can bring your machine to an 8192 tep capacity.

The 31 manual—*basically the same as the 21*—demonstrates other programs requiring increased capacity. What are they? Histograms, General m X n contingency tables, Polynomial Curve fitting *(nth order)* and Multiple Linear Regression *(nth order)*. The 31 Statistics programs are geared for the machine's basic 512 step configuration.

No matter which manual you're into . . . 21 or 31; the documentation is clear and concise. We've made things perfectly clear with such sequences as: PROGRAM DESCRIPTION, EXAMPLES, EXECUTION and PROGRAM STEPS. If you're looking for operating time and error reductions along with versatility and low cost, take a look at our program libraries. It's easier than pounding a lot of keys.



SAMPLING: RABBITS



The rabbits are happier and so are the people dealing in diagnostic analysis. Scarcely fifteen years ago endocrinology (the study of internal secretions) was solidly rooted in the use of "biological" assays. That meant lab animals were routinely used in establishing hormone levels. The old via-the-rabbit pregnancy test is a good example. This was carried out by injecting a urine sample into a live, virgin, female rabbit. The results established the subject's progesterone level as an indication of pregnancy. To get those results, a wait of several weeks was first necessary. The rabbit was then dispatched to see if there was a thickening of the uterine wall, the normal response to progesterone. But these bio-assays always produced a litter of complicated variations: the rabbit might become ill, or die. There was the possibility the rabbit wasn't virgin and the results were always non-quantitative. The only accurate indication of pregnancy was a nine months' wait.

Inaccuracies in checking hormone levels were part and parcel of bio-assay procedures. Not a promising situation for researcher, doctor or subject for that matter.

No Rabbits

But come the revolution. Rabbits were out and radioimmunoassay was in. A new day dawned over endocrinology. Instead of the erratic behavior of rabbits in various states of physiological activity setting the pace, researchers were able to exploit the statistical "law of large numbers". They could depend on 10^{10} molecules in a test tube to behave according to a well-defined chemical and equilibrium theory. Testing for pregnancy could no longer be compared to a roll of the dice.

But radioimmunoassay is just the name of the push for reliability. The study of competitive protein-binding and saturation analysis are working parts of the effort to dampen inaccuracies.

What is radioimmunoassay? In short, it is a form of scorekeeping. It records what happens when new players are introduced to an old game with unvarying rules. The game is binding of antigens (proteins, hormones, etc.) to specific antibodies. Their interaction is unchanging and provides a solid mechanism for measuring antigen levels in a patient's blood sample.

Assume for a moment it's the insulin level of a patient that is to be established. The first step is the introduction of a measured amount of radioactively labeled insulin (antigens) and specific insulin antibodies into a serum sample. Each antibody in the serum (the serum being just the medium) has two "binding sites" or surface areas which will link to an insulin molecule on contact. The introduced insulin and that normally found in the blood will compete for binding sites.

Unbound insulin is separated from bound insulin. The radioactivity levels are then measured. The ratios of labeled insulin bound to total labeled insulin (bound and unbound) are measured, calculated as $^{B}/_{T1}$. By comparing the assay results to a standard curve the exact amount of ambient insulin in the serum sample can be established.

The dose-response curve satisfies a rectangular hyperbola, provided that antigen is present in excess of antibodies. Radioimmunoassay analysis has become the key in ferreting out the level of blood components which have antigen-like properties: proteins, peptide hormones, drugs, vitamins and steroids. It is now a diagnostic tool of the first order—but only through statistics.

Look But Don't Count

Most of the early studies with RIA completely ignored

the use of statistical methods for data processing. This was excusable when dealing with meteoric physiological variation (e.g., response of ACTH[2] or HGH[3] to stress, the midcycle LH[4] peak). However, the need for statistical analysis reappeared for studies of less dramatic physiological effects (e.g., diurnal variations, follicular versus luteal phase LH values, the follicular phase LH rise), and whenever it was attempted to push these new assay methods to their limit.

Better Than Once in a Row?

However, these problems in radioimmunoassay are offset by the ease and economy of replication. In radioimmunossay, it is quite possible to assay a given sample in duplicate (or triplicate) in any one assay and to repeat the analysis for two or even several assays. Thus, by study of the reproducibility of only a small sub-sample of the "tubes" in an assay, it is possible to obtain an empirical estimate of the variance both within and between assays for any desired dose level. This degree of replication was not possible in most bioassays since they were so costly in terms of time, money, and precious samples.

Accordingly, the quality control methods of the clinical laboratory (e.g., as used for blood glucose, potassium, etc.) became applicable to radioimmunoassay. Perhaps the easiest way to perform quality control is to plot the results obtained for a single sample on consecutive assays. If several samples are assayed in each of the assays, then we can plot the total or average of these "Q.C." samples. This procedure provides a measure of the stability or reproducibility of the assay systems. Rules for rejection of an assay as "out of control" can be applied, based on classical industrial quality control methods. A more efficient form of analysis is based on calculating the standard deviation of replicates within assays and also the standard deviation of results from several assays. This method requires slightly more involved computation than in the use of totals or means; however, since these methods are easily performed using either the TEKTRONIX 21 or 31 calculators, the increased efficiency of data utilization is worthwhile.

It All Adds Up

In constructing "quality control" graphs, we can record several assay parameters, viz.: 1) total counts; 2) nonspecific counts; 3) quantity of labeled antigen per tube; 4) specific activity of labeled antigen; 5) slope; 6) intercept (value of dose of unlabeled antigen when B/T is reduced to one-half the value of B/T in absence of unlabeled antigen).

By applying these quality control procedures to several dose levels, we can obtain the relationship between the standard deviation of a potency estimate and the dose level or between the coefficent of variation of a potency estimate and dose level.

Alternately, we can calculate the within-assay and between-assay standard deviation of the response variable ("Y") as a function of dose X or as a function of response level ("Y").

By combining information about the scatter or variance in the response variable with the shape of the dose-response curve, we can predict the magnitude of the "fiducial" or confidence limits for unknowns IF WE MAKE THE ASSUMPTION THAT THE STANDARD AND UNKNOWN DISPLAY IDENTICAL BEHAVIOR. It is the vulnerability of this assumption which makes the use of empirical quality control procedures not only desirable but mandatory. The variance of the response level, either within assays or between assays, is readily calculated by classical methods of analysis of variance (ANOVA) and regression analysis. We have shown, and others have confirmed, that there is an empirical linear relationship between the response level and its variance:

$$\mathrm{Var}(Y) = a_0 + a_1 Y$$

When Y represents the B/T ratio relative to the initial B/T ratio, i.e., Y = B/Bo, then representative values of a_0 and a_1 are $a_0 = 0.0001$ and $a_1 = 0.0004$, although wide variation can be expected, depending on assay conditions.

Predict The Uniformity

As we have just seen, the response variables, B/T, B/Bo, and logit (B/Bo) do not show uniformity of variance over the entire range of doses.

$$\mathrm{logit}\,(B/Bo) = \log_e \left[\frac{B/Bo}{1 - B/Bo}\right]$$

Thus, it would be desirable to derive an expression for the variance of a general response variable Y, either as a function of dose (X) or response Y.

The simplest possible model is: $\mathrm{Var}(Y) = a_0$ However, as noted above, this model fails to describe most real radioimmunoassays. A second model states that the variance of Y is directly proportional to Y: $\mathrm{Var}(Y) = a_1 Y$

Thus, Y is a Poisson or Poisson-like variable. If counting error were the only source of error in Y, then Y would be a Poisson variable.

According to this model, the response variable, $\sqrt{Y}$, should display uniformity of variance. If we count only the bound fraction and if counting error is the only source of variance, then $\mathrm{Var}(B^*) = B^*$ where B^* is the number of counts bound. Accordingly, $\sqrt{B^*}$ will display uniformity of variance. For any fixed specific activity (S), reaction volume (V), and counting time (T), there is a direct proportionality between B^* and $|P^*Q|$, i.e., $B^* = SVT\,|P^*Q|$ where $|P^*Q|$ is the molar concentration of labeled antigen-antibody complex. These considerations lead to the use of $|P^*Q|$ and $\sqrt{[P^*Q]}$ as response variables.

The next most "complex" model for the variance of the response variable is a combination of the preceding two models: We simply speculate that Var(Y) is a linear function of Y, i.e., a combination of a constant and a Poisson-like component.

This is designated Model II: $\mathrm{Var}(Y) = a_0 + a_1 Y$

This model fits well with available data for double antibody radioimmunossays for HGH[4], HLH[5], HFSH[6], for "charcoal" assays for HGH and estradiol, and "Florisil" assays for progesterone and 17a-hydroxyprogesterone. Indeed, for purposes of obtaining a weighting function for empirical data processing, this model is both sufficient and efficient.

The two parameters involved, a_0 and a_1, can be obtained by a minimum of experimentation. We could go on adding additional terms of a power series; i.e., a 3-parameter model could give a better description. However, the linear model was first introduced as a simplification of a more complex but realistic mode, designated Model I.

If we assume that all antibody sites are saturated (which will occur as p^* becomes much larger than q and as K increases), then the dose-response curve for radioimmunoassay becomes a rectangular hyperbola as indicated by the equation: Dose — Response Curve

$$B/Bo = b/(X + b)$$

where b is a constant and X is dose of unlabeled antigen.

Under this simplified set of assay conditions and a number of other assumptions, we can predict the magnitude of error in B/Bo or logit B/Bo by the following equations:

$$Y = \frac{B - N}{B_0 - N} \qquad \mathrm{Var}\,(Y) = \frac{\mathrm{Var}\,(B^*)}{(B_0 - N)^2}$$

$$Y' = \mathrm{logit}\,(Y) = \log_e \left(\frac{1}{1 - Y}\right) \qquad \mathrm{Var}\,(Y') = \frac{\mathrm{Var}\,(Y)}{Y^2\,(1 - Y)^2}$$

$\mathrm{Var}\,(B^*) = (B^*\,v_p\,(1 - Y))^2 + (Nv_p)^2$

variance due to tracer

$+ ((B^* - N)\,v_p)^2$

variance due to antibody

$+ (B\,(\log_e(1 + v_p))\,Y\,(1 - Y)\,(B_0 - N))^2$

variance due to standard

$+ B^*$

counting error

$+ (V_a\,(T - B^*))^2$

misclassification I

$+ (V_\beta\,(B^* - N))^2$

misclassification II

Pipetting error (v_p) is estimated as

$$v_p = \sqrt{\frac{S^2 - \bar{T}}{\bar{T}}}$$

where $\bar{T}$ = mean total counts
S = std. dev. of total counts

This model has proven satisfactory for prediction of weights for weighted least squares linear regression of logit B/Bo vs. log X. It should be mentioned this last model is based on many assumptions and approximations. For model testing information and subsequent model details write: Delmarva Computer Industries, Columbia, Maryland.

Testing, Testing

If you're looking for the tools to carry out the analysis discussed on these pages, we'd like to suggest some: the TEKTRONIX 21 and 31 Statistics Program Libraries. They include a large variety of statistical testing programs. They're made easy to use with a series of easy-to-follow examples. In many cases the results of a statistical testing program (e.g. a ^{2} value) can be easily linked to distribution programs, so the user can compare his calculated test statistics with the appropriate probability distribution.

In addition to statistical testing, the Statistics Program Library also makes you a master of linear, polynomial and exponential fitting; correlation coefficients; means; standard deviations; histograms; random number generation; normal, binomial and Poissan distributions; and analysis of variance.

Improve Your Memory

Both the 21 and 31 have their advantages for statistical applications. The 21 is a lower cost machine, either with its standard 10 data registers and 128 program steps, or with the optional configuration of 256 steps that's recommended for use with the Statistics Program Library.

The 21's program step memory is divided into eight equal sized blocks, each addressable by an f-key (f_0 through f_7). That's an ideal set-up for statistical programs, since it allows each statistical operation to be called by its own key. Each statistical program typically has its own initialization key, a key for more data, and keys for several analyses.

Programs can be conveniently stored on special magnetic cards. The storage is designed so that separate programs can easily be linked together.

You'll learn to look on linkage as a highly useful feature, whether you're running distribution calculations, or doing consecutive statistical tasks on one data set (e.g., for paired variable tests or curve fittings).

Programmability

Both the 21 and 31 can be programmed by the user. You're not limited by any program configuration. If any of the models we've shown here need a little customizing, you can change the programs. If you'd like you can create programs out of spare parts or even write your own from scratch.

The Statistics Program Library for the 21 will cover most applications. For conveniently conducting the analyses we've discussed in this article you may need more elbow room for your programs. That's where the 31 really comes in handy.

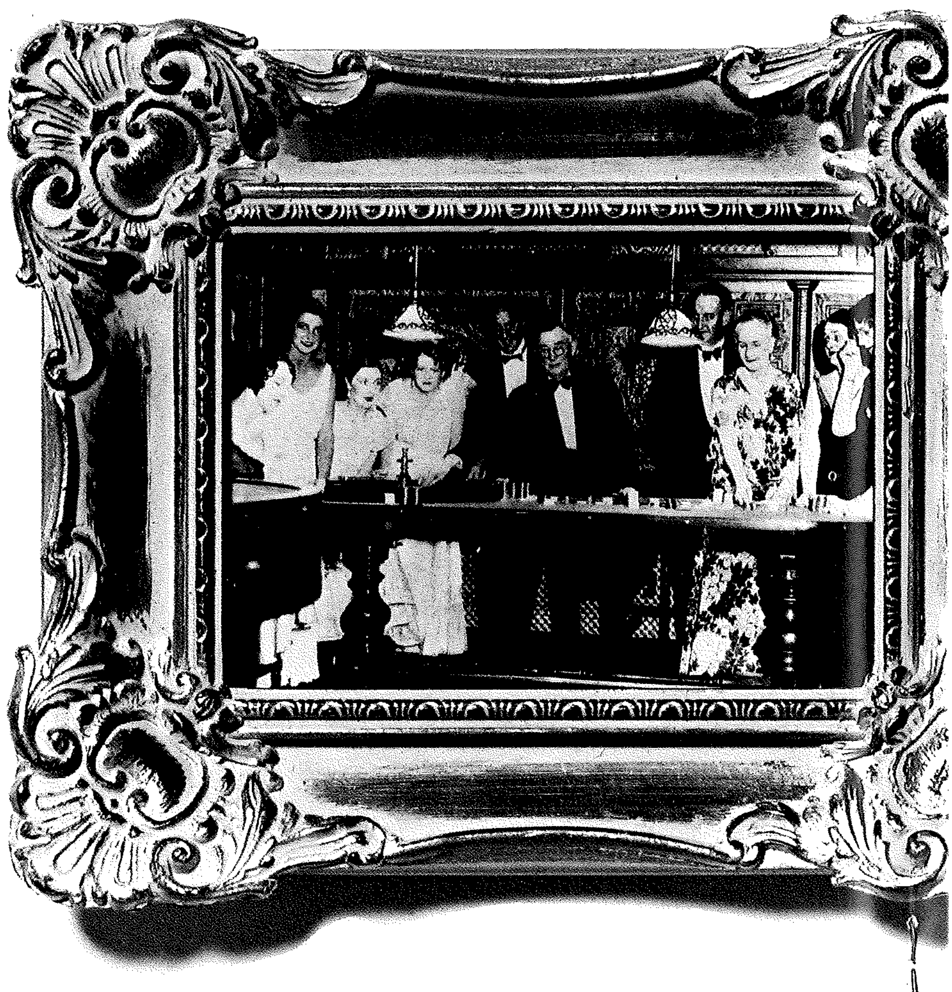
The 31 statistics program library is geared for the basic 31 configuration of 512 steps and 64 R-registers (in addition to the 10 keyed K-registers). (Each register has room for a 10-digit mantissa and 2-digit power of ten.) This basic configuration can be expanded in a number of ways, up to a staggering 8192 program steps or 1000 R-registers.

Stretch Your Limits

Up to 6000 program steps can be stored on one magnetic tape cartridge. So can data register contents (up to 600 registers worth). As in the 21, linkage is an easy option. But on the 31, it's even easier to store a very large program in self-contained chunks, to be automatically loaded and sequentially processed. Furthermore, the contents of data registers can be stored and recalled automatically on magnetic tape. All which offers maximum versatility in biological research work.

Running radioimmunossay research through a TEKTRONIX calculator is like raising rabbits. You can depend on what comes naturally.

1. $\frac{\text{bound antigen}}{\text{total antigen}}$
2. Adrenal Cortical Tropic Hormone
3. Human Growth Hormone
4. Lutenizing Hormone
5. Human Lutenizing Hormone
6. Human Follicle Stimulating Hormone



THE WIZARDS OF ODDS

STATISTICAL SCIENCE AS GAMESMANSHIP

From its inception to the present day, statistical science has kept some of the aura of a great guessing game. Even now, fraught with formulae and surrounded by symbols, it remains an improbable hybrid of efficiency and intuition.

It can be maintained that its origins left it with this road to travel. For the first urge toward pinning down the probabilities—at least in the West—was a gambler's urge.

The first well-remembered reference to orderly probability was in a commentary on Dante's *Divine Comedy,* touching on the probable throws of three dice. A dice-playing Renaissance aristocracy soon turned its attention from Dantean theology to the numerology in the footnotes.

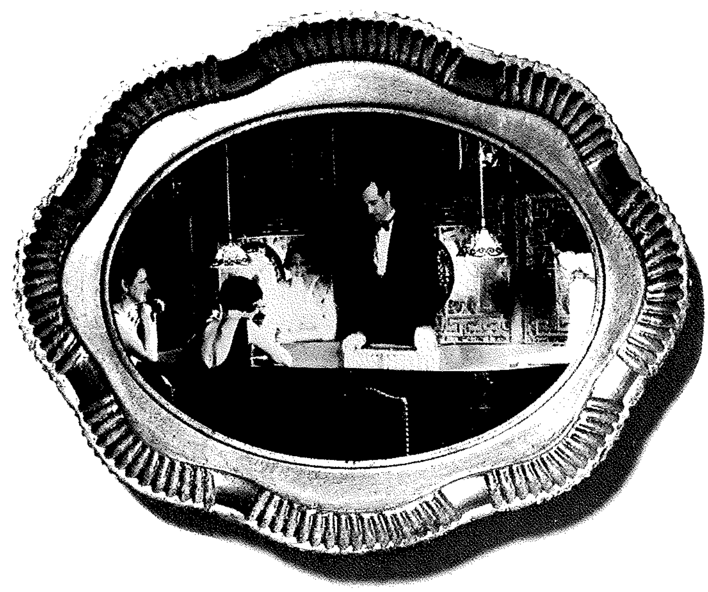
Pushing the odds

Instead of casting the irreligious odds men out, European society began to patronize them. New discourses on probability became a frequent subject for the erudite chitchat of the times.

In 1494, Laca Pacioli first wrote about what was to become the classic "Problem of Points." The problem was how to split up the pot when a game of chance broke up before its natural conclusion. It was an understandably touchy area. Solutions were at least attempted in every probabilistic work worth its binding, for the next two centuries.

A major contribution to the genre was *De Ludo Aleae,* a sort of gambler's handbook, published in 1663. The author, name of Cardan (1501-1576), was an experienced gambler who knew his stuff. As for his theoretical foundations, they were probably sketchy.

In the same era, Kepler and Galileo were setting their scientific sights high into the heavens. But both



looked down long enough to make brief references to our subject. It was their acknowledgement that the game of chance was played beyond the borders of the gaming table.

Still, none of the earlier wizards of odds offered any systematic approach. Nor are their spotty solutions remembered for insight or accuracy.

No, it took the vocabulary of the Rational Man of the 17th and 18th centuries to bring forth the skeletons in their cluttered expository closets. And to rationalize the gambler's fascination.

Blaise of glory

The formal foundation for probability theory was heavily indebted to gamesmanship, though in a rather random way. Blaise Pascale (1623-1662) was one of its unlikely co-authors. He wrote from the reclusive sanctuary whence he had retired at age 25, after having won immortal fame as a mathematician and physician.

Second contributing factor was Pierre de Fermat (1601-1665), best known for his theory of numbers. Never published, Fermat left posterity only his letters to such heavy pen-pals as Pascal and René Descartes who shared his interests.

But it was an unremembered Chevalier de Méré who catalyzed the outcome. He presented the retreating Pascal with various quasi-mathematical problems, all of interest to gamblers (such as de Méré), and including the "Problem of Points." That started a volley of letters not soon to be forgotten in statistical annals.

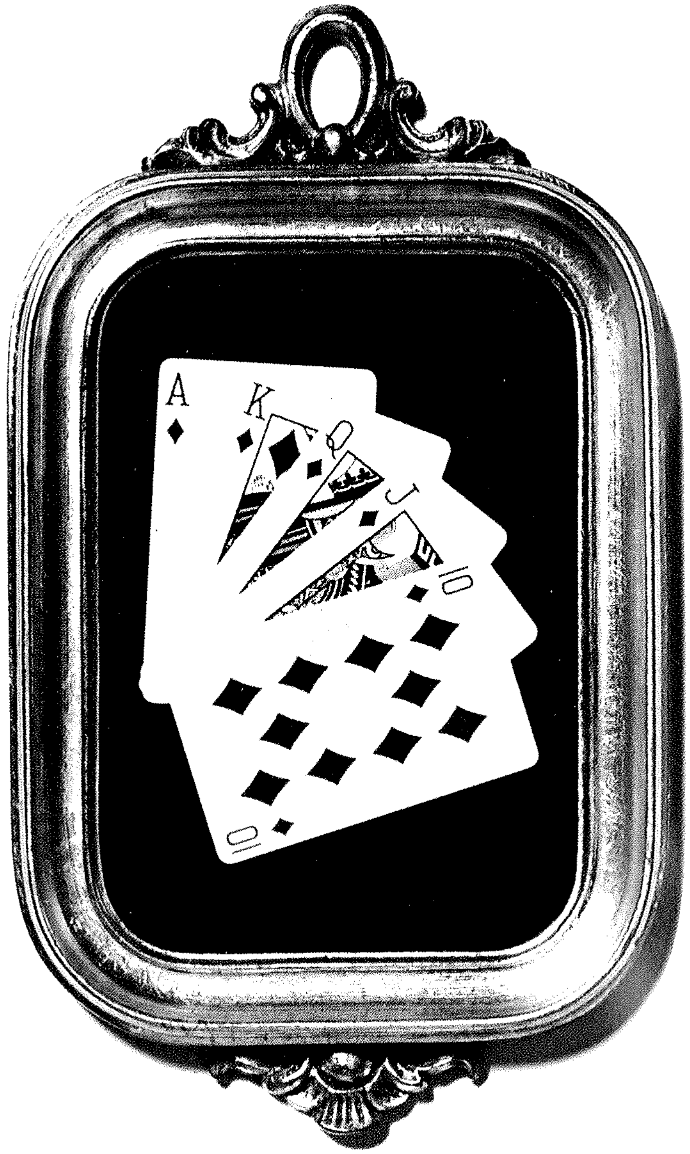
De Méré was a gambler with acknowledged mathematical ability (certified by Leibniz himself). Pascal was a mathematician profoundly interested in philosophical questions of knowability. By 1654, with indirect inputs from Fermet, what had been intellectual diversion was reaching toward generalized mathematical concepts of great scope.

That should have launched probability as a science. But the calculus of Newton and Leibniz diverted energy from probabilistic inquiry for several decades.

Then in 1692, an English translation of an ambitious Dutch tract appeared, brimming with mathematical advice on various card and dice games. Forthwith, avarice threw the spotlight back on statistics theory.

This Huygens treatise intrigued, among others, the Swiss mathematician Jacques Bernoulli (1654-1705. His *Ars Conjectandi,* published posthumously in 1713, became the first book devoted purely to probability.

The *Ars* put an important stamp of approval on the "conjectural art." In it, Bernoulli, a respected mathematician, suggested that probability be applied to civil, moral and economic areas, not just to the gaming table. He first hinted at the importance of inverse probability. He discoursed on the role of permutations (a word he coined) and combinations in Nature's bounteous variety, and analyzed man's imperfect perceptions of same.

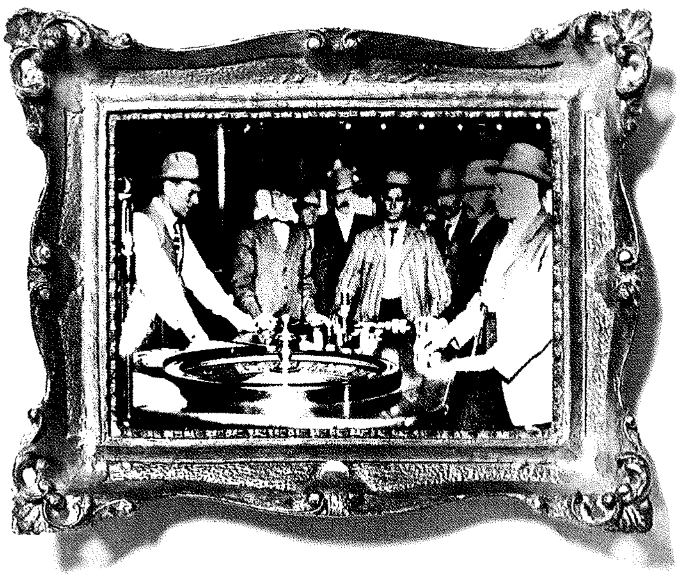


He also took a few irresistible shots at games of chance. And he began defining terms, a habit of statistical theorists that has since become a mixed blessing. Consider Bernoulli's "morally certain," a title used to define cases where the "degree of certainty" is .99, or even .999. A convenience to judges, thought J.B.

But in the next breath he worked very analytically with a binomial expansion to compare (very roughly speaking) the sum of terms in the expansion so far to the sum of terms left, at a given term n. (This method of finding a probability ratio, though precocious, has been guaranteed to ruin any insurance company that tries it.)

Beat at their own game

Pierre-Rémond de Monmort (1678-1719) next turned the tables on the whole subject. He wrote a



probability essay on "jeux de hazards" that left non-mathematical gamesters in the dust.

A contemporary statistical saint, Abraham de Moivre (1667-1754), thought little of Montmort's work, and de Moivre has had the greater influence. But perhaps de Moivre's judgment, like his income, was dependent on his sideline of solving mathematical problems for wealthy English patrons. Many of them, no doubt, were more concerned with odds than with abstractions.

De Moivre did publish up into the realm of infinite series. And he chided readers to learn "Vulgar Arithmetic" and the "bare Notation of Algebra" if they wanted to keep up with probability.

Furthermore, it's to de Moivre that we owe the 1733 discovery of the normal curve, the darling of psychologists and bane of students everywhere.

Actually, this story of probability's environment with gambling ought to end with de Moivre (Though it doesn't). He pointed out that the limiting nature of his probability theory showed error dwindling as number of observations increased.

To his satisfaction, the infinite variety in creation bred the certainty of a divine order.

Bravo, de Moivre. Out of the footnotes and back to Dante at last.

Tell the players by their programs

But a cursory examination shows statistical practices still dabbling in gambling, now on a global scale. Projections determine national policies. Samplings determine projections. And God-knows-what kind of statistical theories get us from sample to certainty.

Consider statistical code cracking, an important probability side-line. On it hinges the security of whatever national or corporate wisdom may finally be communicated. Based on the probable frequencies of letters in a message, it succeeds in making secret-sending a game of no-score ties.

And consider further that much of the painfully verbose probability theory with which students are still stuck was, in fact, done to cover various computational lacks. With neither calculators nor computers, some of the finest minds of Europe spent their time hypothesizing formulas to approximate what might result—if they could only figure out the complex terms of converging series.

In 1671, a practical scholar published a *table* of "scientifically determined" annuity purchase prices. Had he published his formulas, no one would have taken the time to calculate the values. Even though it would have meant profits to do so.

Other luminaries, with only scholarly points at stake, cried for figuring lackeys to come forth and get them out of their pre-calculation uncertainties. But cries were plenty, and human calculators were few. The rewards in the rarified non-gamblers' circle were simply too small, and the odds against glory too great.

Pity, then, that today statistical calculations are being made so easy. Now a man with nothing at all at stake can have his coefficients, curves, and prognostications at his fingertips.

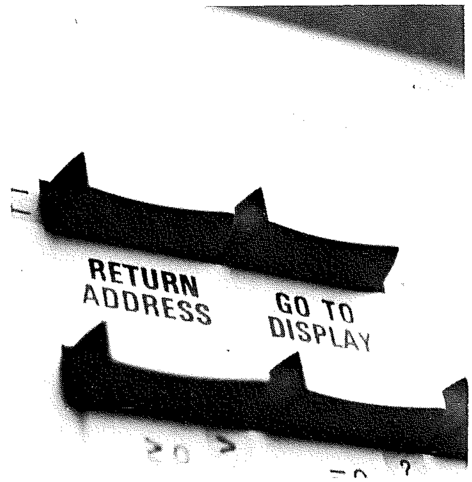
It marks the end of a sporting era. Such convenience can only succeed, where scholars failed, in finally taking the gamble out of statistics.

SUBROUTINES GETTING THE PICTURE

So you've got that strange sensation again? Somehow you know you're doing something you've done before. Ever hear of Deja Vu? That's what Webster calls the act of reliving a pást experience. Deja Vu. That may be what you've got. If you're a 31 programmer the chances are especially good. You can expèrience it as you key the same instruction into your 31 for the 'nth time. Sound familiar? If so we'd like to suggest something. Subroutines. What is a subroutine? Just that—actually a sort of mini-program.

A subroutine is useful whenever you've got a task repeatedly used in the same program. That same subroutine could also be put to use in a variety of programs—but it'll stand alone. Subroutines can be called from the keyboard in addition to their normal program execution. This keyboard calling makes it easy to write conveniently used modular programs. This type of program construction permits fast changes in configuration. Consequently, debugging is also simplified. For future reference any subroutine, no matter what its function, is easily stored on a magnetic tape. The recorder is built right into the 31. This way when new applications present themselves you won't need to invest your time rekeying.

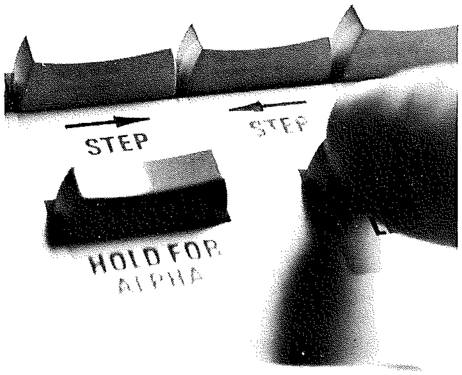
"debugging is also simplified"



Getting there and back. To use the subroutine in a program, a method of getting to it as well as back again must be provided. It depends on your program and machine configuration as to which of several alternative linking techniques you use. "Linking refers to getting at a subroutine." One simple method is to put your subroutine at the beginning of the memory, which requires only a stroke on the "start" key for execution. Another method is to place the subroutine at a specified spot in the calculator and use the "go to" command to bring it into use. You could also store its location in a register and later recall to display and simply touch the "go to display" key to program the execution of this task.

"linking refers to getting at a subroutine"

Probably the most convenient way of linking is the labeling technique. Simply put in a label key and any symbol (a symbol is nearly any key in the calculator) as the first two sequences of the subroutine. When that symbol is used with the label key,



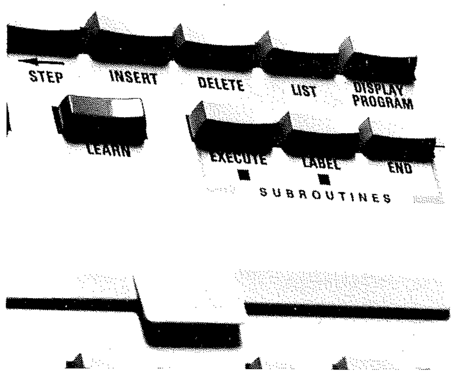
it, in effect, becomes the name of the subroutine. Execution begins by simply pressing "exec" and the "name" of the routine.

"you are, of course, the best judge"

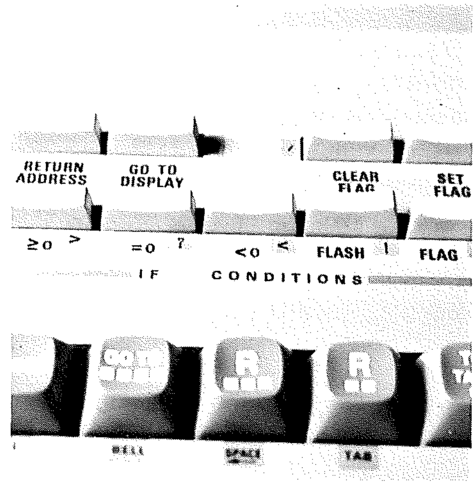
You are, of course, the best judge as to which linking technique to apply to your subroutine as well as when and where to use it.

A Link with the Past. When your subroutine has done its job it's easy to get back to the main program, especially with the TEKTRONIX 31. Each of the linking techniques automatically loads the address of the next sequential instruction of the calling program in the return address register. When your subroutine is finished, simply recall the return address and execute the "go to display" command.

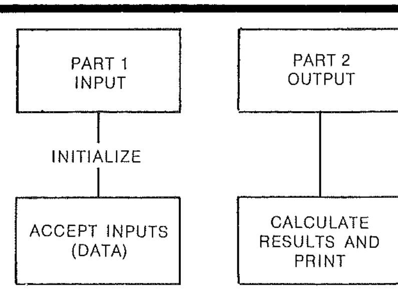
Nesting Sub-routines. This refers to the technique of having one sub-routine call another. Remember that each time sub-routines are linked, the return address register is updated. This process of updating the return address register destroys any previous address. That means to "nest" sub-routines successfully, you'll want to



preserve the return address in some other register prior to calling the next subroutine. This is accomplished by storing the return address in a different register. When you want to return to the main program, you "recall" that register, instead of the "return address" register, to return to the main program. By using unique "save" registers, subroutines can be nested to as many levels as you think you'll need.



Raise the Flag. Now that you know the rules for getting to and from sub-routines, let's get down to bare wood with an example. The TEKTRONIX Statistics Program Library includes a Histogram program that reads values into the registers then displays them via the 31 printer. The program has a two part format as shown below.
Part 1. Input: Initialize, accept inputs (DATA)
Part 2. Output: Calculate, results and print.



Part one, the initialization step, clears the registers, asks you for the minimum and maximum value of x's with the width of the cells. Part one also accepts inputs value and adds 1 to the counter representing the particular cell in which that value falls. Its next sequence is to count that input. It is now ready to accept another data value. This step in the program will continue as long as you enter values and press "continue".

When you want to discontinue input and calculate the results, press the keys to begin the output part of the program.

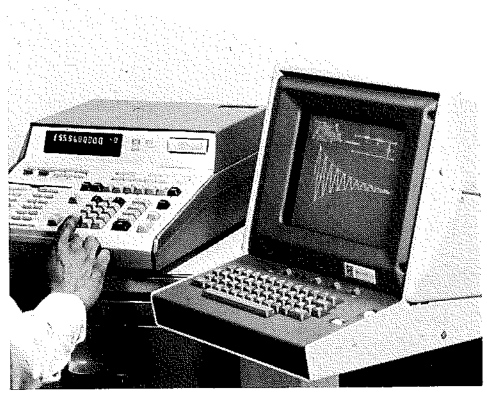
Each cell is read in separately and the % of total is calculated and printed.

And if you're manning a 31/10 you've got the power to demonstrate histograms graphically.

Let's change the output instructions so that instead of printing the values, it

"draw a histogram on the TEKTRONIX 4010 Graphic Terminal"

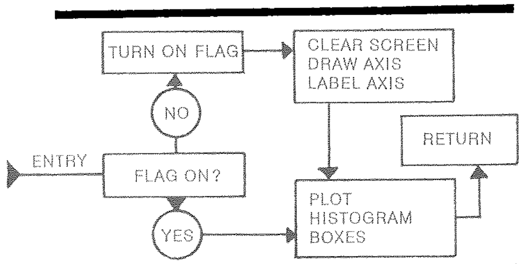
calls a subroutine to draw a histogram on the TEKTRONIX 4010 Graphic Terminal. It's no big deal. To alter the program, simply change the "print" command to an "execute 1" linking command such as we discussed earlier. This way instead of printing the percentage of each box or cell, the program will execute a subroutine labeled '1'. This subroutine has two parts.



Part A: To clear the screen, then draw and label the axis.

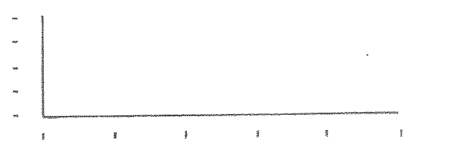
Part B: To plot and illustrate the individual histograms boxes as they are developed.

Part A need only be executed once during the first subroutine operation in each program. It can then be by-passed in all succeeding calls to the plotting subroutine. We can do this with the flag check provided on the TEKTRONIX 31. The subroutine flow will look like this:

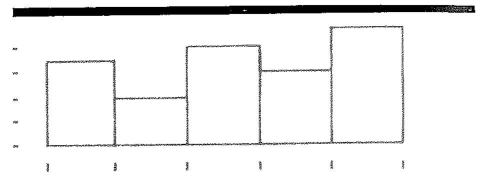


If the flag is "off" the first time the subroutine is called, the calculator will put Part A in operation. This action clears the screen, establishes the X and Y axis, labels them and then turns on the flag. Your calculator then executes Part B, the only section necessary in subsequent use of the subroutine. Upon its completion, your calculator returns to the main program.

At the conclusion of Part A where the flag is "off" the screen of the 4010 will look like this:



When all the cells have been displayed the screen will present the histogram data in an easy to read graphic histogram.



Clear Across the Screen. Now that you've seen the histogram subroutine results, you should know how its steps were put together. At the start of the first subroutine in a 31 program, the test for flag (the "if flag" command) will show it to be in the "off" position. As we discussed, the calculator will then initiate Part A. As you'll recall, the first act is to clear the 4010 screen. Easily done. Sending a four character command string to the 4010 will erase the screen and put the terminal in a mode to begin processing. You've now got a clear field for your axes. The "x" axis is along the bottom of the screen while the "y" axis will run vertically. For any point on the screen, it's necessary to provide both an "x" and "y" coordinate. To draw the "x" axis, therefore, will require four coordinates—an x and y for the start and an x and y for the end of the axis. Establishing the starting coordinate is easy. On the 4010 there are 1024 points on the screen's "x" axis while the "y" axis runs up to 780 points. Selection of the minimum "x" and "y" coordinate for the "x" axis is an arbitrary decision,

First Class
Permit No. 61
Beaverton
Oregon

BUSINESS REPLY

No postage stamp necessary if mailed in the United States

Postage will be paid by

TEKTRONIX, INC.
P.O. Box 500
Beaverton, Oregon 97005

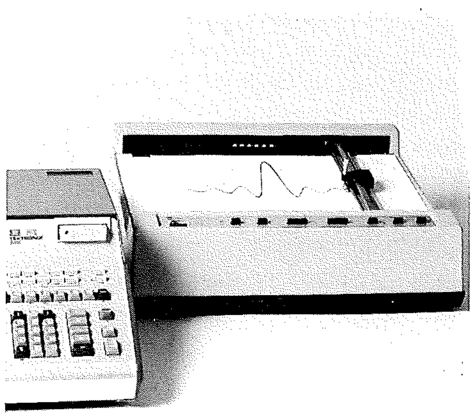
Attn: Jim Buchanan

a line or "x" axis. Using the same bag of tricks will produce a "y" axis. The starting coordinate is identical to that of the "x" axis. Move to that point. You can now be just as arbitrary in picking out the maximum "y" axis as you were with that of the "x" axis. Let's say the number 700 is appealing. You can now draw a line to that point.

Increments by the Number. Since the "y" axis always represents 100%, just locate that number at its maximum and label it. Numbering the "x" axis is a little different. First, you'll recall that you've got an "x" axis range of 800 points (axis maximum of 1000 less axis minimum of 200). Within your histogram program is a register that contains the number of cells in the histogram. Simply divide the number of "x" axis points by the number of cells to get the increment length along that axis.

Also, within the program are the minimum and maximum data values. The range of data values divided by the number of increments will produce the actual numbers to be printed along the "x" axis. To form those labels, move to each location and print those numbers. You know the number of boxes and the width of each increment.

Left Side, Right Side, All Across the Screen. The second section or Part B, is really the bread and butter of this subroutine. This is where you'll start in all succeeding subroutine use because the groundwork has been laid and the flag is "on". This is the place to plot the histogram's bars or boxes. Here's how. You've got the minimum "x" coordinate and the minimum "y" coordinate in registers. The first step is to move to the location on your screen referenced by those registers. Next, to locate the "y" coordinate that represents the top of the box, simply call up the register which is 700 less the 200) and add this figure to that "y" minimum of 200. Move to the minimum "y" coordinate then draw vertically to that first figure. You've now got the left wall of your first box. To draw the top of your box you've got to know how far to the right to run your line. To come up with that answer, add the "x" increment you've already established to the register that contains the "x" minimum. Draw to that point. Last, we draw back down to the "y" minimum. That's the right wall. That location, which is called the new "x" minimum, is starting point for the next histogram box. Now that you've laid the groundwork, each time you need your histogram subroutine, a touch of your execute key and label will put it into action.



"a touch of your execute key"

The 31/10 Statistics Program combination provides a powerful push for your ideas. We hope the concepts presented in this article will help enhance your reporting and representation skills while using those ideas. They're a sure cure for Deja Vu.

but as you might expect it is generally located in the lower left of the screen. For this example, you could use 200 as the value of that coordinate for the start of the "x" and "y" axis. You should then move to that coordinate. A move, incidentally, is the process of shifting the drawing mechanism to a particular coordinate without making a visible mark.

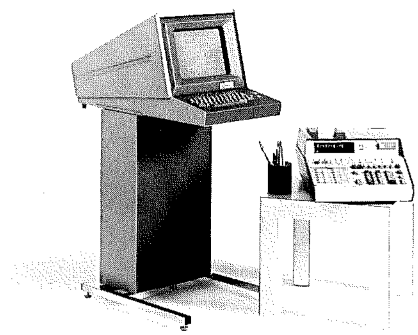
Next, pick an ending coordinate for the "x" axis. If the minimum "x" is 200, the maximum "x" can just as easily be an arbitrary 1000. Because you haven't made any vertical moves, the "y" of the second "x" and "y" coordinate is still the same. Now that

"now that you've got both coordinates located"

you've got both coordinates located on your screen you can run a line or "draw" between them. You've now got a line or "x" axis. Using the same bag of tricks will produce a "y" axis. The starting coordinate is identical to that of the "x" axis. Move to that point. You can now be just as arbitrary in picking out the maximum "y" axis as you were with that of the "x" axis. Let's say the number 700 is appealing. You can now draw a line to that point.

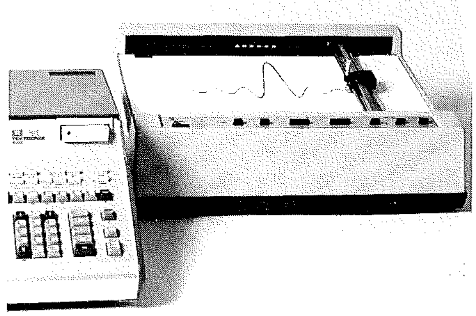
Increments by the Number. Since the "y" axis always represents 100%, just locate that number at its maximum and label it. Numbering the "x" axis is a little different. First, you'll recall that you've got an "x" axis range of 800 points (axis maximum of 1000 less axis minimum of 200). Within your histogram program is a register that contains the number of cells in the histogram. Simply divide the number of "x" axis points by the number of cells to get the increment length along that axis.

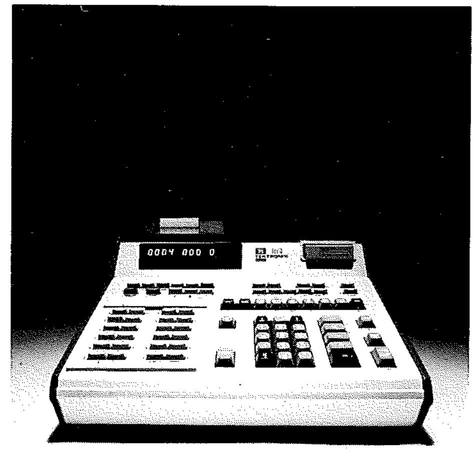
Also, within the program are the minimum and maximum data values. The range of data values divided by the number of increments will produce the actual numbers to be printed along the "x" axis. To form those labels, move to each location and print those



numbers. You know the number of boxes and the width of each increment.

Left Side, Right Side, All Across the Screen. The second section or Part B, is really the bread and butter of this subroutine. This is where you'll start in all succeeding subroutine use because the groundwork has been laid and the flag is "on". This is the place to plot the histogram's bars or boxes. Here's how. You've got the minimum "x" coordinate and the minimum "y" coordinate in registers. The first step is to move to the location on your screen referenced by those registers. Next, to locate the "y" coordinate that represents the top of the box, simply call up the register





with the percentage value of that particular cell. Multiply that percentage times the range of the "y" axis (500 which is 700 less the 200) and add this figure to that "y" minimum of 200. Move to the minimum "y" coordinate then draw vertically to that first figure. You've now got the left wall of your first box. To draw the top of your box you've got to know how far to the right to run your line. To come up with that answer, add the "x" increment you've already established to the register that contains the "x" minimum. Draw to that point. Last, we draw back down to the "y" minimum. That's the right wall. That location, which is called the new "x" minimum, is starting point for the next histogram box. Now that you've laid the groundwork, each time you need your histogram subroutine, a touch of

"a touch of your execute key"

your execute key and label will put it into action.

The 31/10 Statistics Program combination provides a powerful push for your ideas. We hope the concepts presented in this article will help enhance your reporting and representation skills while using those ideas. They're a sure cure for Deja Vu.

CROSSPOLLENIZING

Earlier in this issue we said that calculations will reflect not only our point of view but yours. This section is the focal point for the direct exchange of ideas. It is the forum for comments, suggestions and occasional bronx-cheers that all dynamic publications can expect. If you have something to say let us know. We can't always promise to print it but we can guarantee its effect will be felt. The Editor

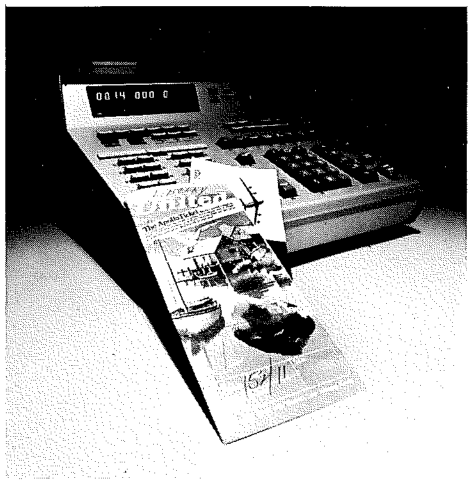
How to Gamble If You Must: Inequalities for Stochastic Processes
Lester E. Dubins, University of California, Berkeley; and **Leonard J. Savage,** Yale University: Series in Probability and Statistics.
1965, 249 pages (017872-0), $12.75.
Contents: Formulation of the Abstract Gambler's Problem. Strategies. Casinos with Fixed Goals. Red-and-black. Primitive Casinos. Some General Principles. Houses on the Real Line. Three Particular Kinds of Casinos. One-lottery Strategies. Fair Casinos. The Scope of Gambler's Problems.
McGraw-Hill Book Co.,
1221 Avenue of the Americas
New York, N.Y. 10020

Statistics in Biology, Volume One
Chester I. Bliss, Connecticut Agricultural Experiment Station and Yale University. 1967, 537 pages, (005895-4), $18.00.
The first of a three-volume series that describes the statistical methods useful in biology and natural science. This volume covers material ranging from the Binomial Distribution through the Analysis of Fourfold Tables, Normal Distributions and Linear Regression. Presents statistical methods in a form suitable for solution on a modern desk calculator. 150 worked examples and about 120 exercises based on real data from wide range of biological disciplines. Reference tables of statistical functions. No prior knowledge of statistics necessary.
Contents: A Taste Experiment. The Binomial Distribution. The X^2 Distribution. Analysis of Fourfold Tables. The Normal Distribution. Numerical Analysis of Normal Samples. Provisionally Normal Distributions. Intervals in Statistical Estimation. Comparison of Two Groups. Comparison of Several Groups. Simple Experimental Designs. Subsampling in Relation to Experimental Design. Linear Regression.
McGraw-Hill Book Co.,
1221 Avenue of the Americas
New York, N.Y. 10020

The Biological Efficiency of Protein Production. J. G. W. Jones, Editor.
Senior Research Fellow, Department of Agriculture, University of Reading and Principal Scientific Officer, Ecology Division, Grassland Research Institute
6x9, c. 400 pp., July. 20179 9 $21.00.
Contents: A wide range of the methods and forms of production of protein in plants, animals and industrial processes are reviewed in this report of a recent symposium. It holds interest for persons in many disciplines, including ecology, microbiology and agriculture.
Cambridge University Press
32 East 57th St.
New York, N.Y. 10022

Preparation For Basic Statistics: A Program for Self-Instruction
Virginia A. Clark, University of California and **Michael E. Tarter,** University of California, Irvine, California College of Medicine.
1968, 224 pages (011130-8), $7.50; Soft cover (011131-6), $5.25.
This programmed book was designed to provide a background in mathematics needed for a first semester basic statistics course at either a junior college or a four-year college. This book should be useful to the student who is weak in mathematics, but who needs to take an applied statistics course as given in psychology, sociology, business, education, public health, and other departments. The book is meant to be used as a supplemental text. The level is aimed at the student who usually does poorly in a basic statistics course because of a low aptitude or knowledge of mathematics, and who needs remedial help. The authors have attempted to present topics from algebra, trigonometry, and geometry which are needed to understand statistics, and at the beginning of each chapter is a discussion of where the material in that chapter is used in statistics.
McGraw-Hill Book Co.,
1221 Avenue of the Americas
New York, N.Y. 10020

CROSS

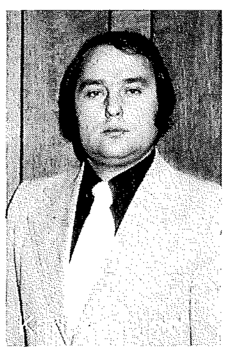


NUMERO UNO

Need help with a lot of pressing matters? That was the case with Loomis Products of Levittown, Pennsylvania. They're manufacturers of Isostatic, Isomatic and Hydraulic Extrusion presses.

As you may suspect, Loomis people have a lot of design analysis going. Besides developing standard off-the-shelf machines, they're big on custom built equipment—the kind of thing that keeps engineers up late. All this design work means people have to put together a lot of mathematical pictures of the stresses in mechanical and hydraulic systems. To do it right requires wading through a lot of formulas. That takes time and time has a way of being reflected on a price sheet. Pushing up engineering process efficiency is one way to push down prices and competition. That's why Loomis was looking for an improvement in numbers handling. They recently found one.

When V.P., Al Ganss was returning from the West Coast, he found himself sitting next to a Tektronix sales engineer, Larry Weese, who was just coming back from the unveiling of Tektronix's new calculator line. Even at 30,000 ft., salesmen are salesmen, so conversation soon centered on possible TEKTRONIX 31 applications at Loomis.



Back at the plant, Textronix's local representative, Dick Sabella soon arrived with a shiny new 31 under his arm. After a demonstration session with Al and President, Karl Kahlefeld, the sale was made. Loomis had found its number handler and Tektronix its first 31 Calculator customer.

What will Loomis be doing with their 31? To answer that, the term "Isostatic" should first be defined. Meaning "subjected to equal pressure from all sides". It refers here to the way Loomis presses create parts out of powdered ceramic, metal or plastic. The process creates high density products which usually require no finishing and with their high tolerances and quality are superior to their cast, forged or machined counterparts. Isomatic presses do exactly the same thing but on a high speed production line basis. Loomis' other line, hydraulic extrusion presses squeeze out a variety of materials using the toothpaste out of the tube concept.

The presses range in capacity from 20 to 400 tons. All require design and analysis. But it's not a one shot per model deal. 90% of Loomis' work is custom built stuff—either from scratch or modifications of existing equipment. That means a lot of engineering going. The whole thing is a natural for the 31. Isostatic pressures, tangential and equivalent wall stresses, fatigue and vessel burst stresses, safety factors, differential thread loading, and thread shear stresses. All these calculations and more are going to be streamlined as they go through Loomis. Thanks to the 31.

In addition to a plethora of engineering numbers, the 31 will be put to use in the Accounting Department. The 31's versatility will also be of use in Loomis' sales and marketing research.

Press on, Loomis, press on.

TEKTRONIX®

BULK RATE
U.S. POSTAGE
PAID
Tektronix, Inc.

Tektronix, Inc.,
Information Display Division
P.O. Box 500
Beaverton, Oregon 97005
Telephone (503) 292-2611
or Tektronix Datatek N.V.
P.O. Box 7718 Schiphol Airport
The Netherlands

Calculations, Number 1, Dec. '73
As the name implies, this quarterly magazine discusses the which, why and how of putting numbers together. The Information Display Division of Tektronix, Inc. produces it in the hope that it will be useful as a problem solver as well as an information center. This issue is the result of time and effort from:
• Ellie Bloomfield, writer • Leroy Nolette, software engineer • Larry Jackson, photographer • Ted Hoff, editor • Frank Roehr, designer.
Address all correspondence to
Jim Buchanan, Devil's advocate and Calculator advertising supervisor,
P.O. Box 500
Beaverton, Or. 97005

A-2793