It is difficult to check the formula experimentally from radiation patterns because of interference between the field radiated by the surface wave and energy radiated directly from the device which launches the surface wave onto its reactive surface. As yet there is no simple method of calculating the amount of energy coupled to the surface wave compared with that radiated directly from the launcher feed.

To circumvent the above problem we have considered the radiation pattern of a slow hybrid wave travelling in a rectangular corrugated waveguide (Fig. 2). The analysis, described by Bryant[4] and Brillouin,[5] shows that, for the dominant slow mode, the field in the y direction is given by

$$E_y = C\cosh(\alpha y)\cos\left(\frac{\pi x}{A}\right)\exp\{j(\omega t-\beta z)\} \qquad (3)$$

and

$$\alpha\tanh\left(\frac{\alpha B}{2}\right) = -\beta_1\tan(\beta_1 H) \quad . \; . \; . \; . \quad (4)$$

where $\beta_1{}^2 = k_0{}^2-\pi^2/A^2$ and $\beta^2 = \beta_1{}^2+\alpha^2$.

For no reflections at the open end of the corrugated waveguide, application of radiation formulas over the aperture gives the normalised E plane radiation pattern as

$$E_n(\theta) = \left(\frac{\beta k_0+\beta_1{}^2\cos\theta}{\beta k_0+\beta_1{}^2}\right)\left\{\frac{\alpha}{\sinh(\frac{1}{2}\alpha B)}\right\}$$

$$\times\left\{\frac{k_0\sin\theta\cosh(\frac{1}{2}\alpha B)\sin f+\alpha\sinh(\frac{1}{2}\alpha B)\cos f}{k_0{}^2\sin^2\theta+\alpha^2}\right\} \qquad (5)$$

where $f=\frac{1}{2}k_0 B\sin\theta$.

Now if the waveguide is slowly flared out to form a rectangular pyramidal horn, so that at the horn aperture $\frac{1}{2}\alpha B$ is much greater than 1, the electric field around the H plane centre of the horn given by eqn. 3 is simplified to

$$E_y = D[\exp\{\alpha(y-B/2)\}+\exp\{-\alpha(y+B/2)\}]$$

$$\exp\{j(\omega t-\beta z)\} \qquad (6)$$

where D is a new constant.

Eqn. 6 represents a surface wave travelling in the z direction on each corrugated surface of the corrugated horn, and the α of eqns. 6 and 1 are the same.

For $\frac{1}{2}\alpha B \gg 1$, $\beta_1 \simeq k_0$ and eqn. 5 is modified to

$$E_n(\theta) = \left(\frac{\beta+k_0\cos\theta}{\beta+k_0}\right)\left\{\alpha\frac{(k_0\sin\theta\sin f+\alpha\cos f)}{k_0{}^2\sin^2\theta+\alpha^2}\right\} \qquad (7)$$

The second bracket of eqn. 7 can be represented by two line sources along the corrugated edges at $y=\pm B/2$ with individual radiation patterns of

$$F_1(\theta) = \frac{\alpha}{\alpha+jk_0\sin\theta}$$

and

$$F_2(\theta) = \frac{\alpha}{\alpha-jk_0\sin\theta}$$

and we see that

$$F_1(-\theta) = F_2(\theta) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

Hence, from eqns. 6, 7 and 8 the normalised E plane radiation pattern of a single surface wave is given by

$$E_n(\theta) = \left(\frac{\beta+k_0\cos\theta}{\beta+k_0}\right)\frac{\alpha}{\alpha-jk_0\sin\theta} \quad . \; . \; . \; . \quad (9)$$

which is the same as eqn. 2.

The radiation pattern of eqn. 7 is useful because the radiation comes from a finite aperture and there is thus no interference by radiation from a secondary aperture. To verify the formula, a corrugated rectangular pyramidal horn was constructed 12 cm long, with an aperture 9·45 × 7·10 cm and having slots 0·725 cm deep and 0·178 cm wide. The teeth were 0·058 cm thick.

The measured E plane radiation pattern at a frequency of 7·5 GHz is shown in Fig. 2, together with the pattern calculated from eqn. 7. The agreement is good at this frequency, but deteriorates as the frequency rises owing to the combined effect of phase error (although the calculated patterns can be corrected for this) and the effects of the 1·5 cm-thick E plane edges which, acting as flanges, can appreciably widen the main beam.[6, 7]

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USE OF HIGH-RELATIVE-PERMITTIVITY MATERIALS IN HIGH-DELAY COAXIAL LINES

Indexing terms: Coaxial cables, Delay lines, Dielectric materials

The delay characteristics of a common type of electromagnetic delay line are considered as both an unshielded line and a shielded line filled with materials of high relative permittivity.

It is possible to produce a v.h.f. delay line by means of an effective slow-wave structure consisting of a helix of coated wire wound on a plasticised ferrite core. With 30 turns/cm and a diameter of 0·3 cm such a structure will produce a delay of 0·18 μs/m for a pulse injected into the line and reflected from the open end. This compares with a delay of 0·008 μs/m obtained with a conventional 50 Ω coaxial line. However, such a system is lossy owing to radiated fields and hence is sensitive to stray capacitance effects, and both the loss performance and the delay time have been improved by enclosing the helical structure in a coaxial shield. The increased delay with the helix is attributed to the increased distributed inductance per unit length of the composite line.

It has been considered possible to increase the delay time by filling the space between the helix and its shield with a material of high relative permittivity. A possible structure is indicated in Fig. 1 and, because of the configuration in the

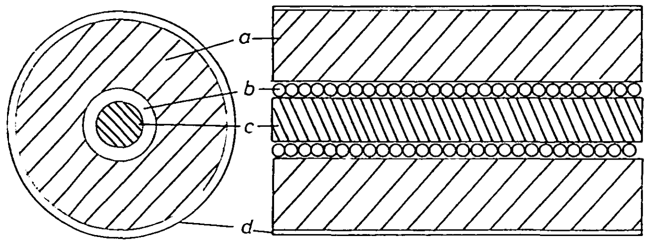

Fig. 1 *Delay line with material of high relative permittivity*
a Material of high relative permittivity
b Coated wire spiral
c Plasticised ferrite core
d Shield

composite line, the fractional increase in capacitance which results from the presence of the high relative permittivity will not be as large as would be observed in a conventional line. It is, of course, unlikely that a simple TEM mode is being propagated in the system shown, and this is confirmed from the results obtained by enclosing the core in shields of various radii, as shown in Table 1. The distributed capacitance, and hence the delay, was found to decrease with decreasing radius, in direct contrast to the result which would be expected for the electric-field configuration of a conventional 50 Ω coaxial line.

Table 1 VARIATION OF DELAY AND IMPEDANCE FOR DELAY LINE WITH AIR FILLING BETWEEN SHIELD AND CORE

Diameter of shield	Delay
mm	μs/m
No shield	0·18
11·0*	0·23
6·0	0·21
4·0	0·17

*Dimensions of a conventional coaxial line

The reduction in the delay time with decreasing radius was also accompanied by lower losses.

A low value of effective relative permittivity for the high-relative-permittivity material may result from both the field configuration in the line and the presence between the dielectric and the conductors of air gaps, which are particularly difficult to eliminate in the composite line. The combined effects may be seen clearly from the effective relative-permittivities shown in Table 2. (The necessary porosity of the ceramic barium titanate would also contribute to a reduction in the value of ε_{eff}.)

Table 2 EFFECTIVE RELATIVE PERMITTIVITY WITH CONVENTIONAL COAXIAL LINE AND COMPOSITE LINE-SHIELD DIAMETER 1·1 cm

Dielectric	ε_{eff}, 50 Ω coaxial line	ε_{eff}, composite line
Air	1	1
Polythene	2·4	1·97
Barium titanate†	80	3·2

† The ceramic $BaTiO_3$ had metallised contacts on the outer surface in the composite line, and on both inner and outer surfaces in the conventional line

The effect of porosity of the dielectric was particularly marked when attempts were made to obtain maximum filling by means of fine powders of rutile and barium titanate. The values of ε_{eff} (and the corresponding delay) were in fair agreement with basic theory and are shown in Table 3.

Table 3 EFFECTIVE RELATIVE PERMITTIVITY FOR COMPOSITE LINE

Material	Effective relative permittivity	Relative permittivity of single crystal
Ceramic $BaTiO_3$‡	3·2	1000
Powdered $BaTiO_3$§	2·9	1000
Powdered rutile§	2·6	100

‡ Porosity = 9% § Porosity = 40%

Table 4 DELAY TIME AND IMPEDANCE FOR A NUMBER OF COMPOSITE LINES

Line	Delay	Impedance
	μs/m	Ω
No shield	0·18	2250
Shield and air	0·23	2260
Shield and polythene	0·35	1500‖
Shield and $BaTiO_3$	0·42	1260

‖ Commercial line

In conclusion, it has become apparent that the composite line cannot be considered as having only coaxial-line characteristics. The absence of any dramatic increase in the delay time for the composite line when a high-relative-permittivity material is inserted between the core and shield can only be accounted for partially by porosity for powders and by gaps for ceramics. However, the changes in the characteristics by the inclusion of barium titanate in a composite line are sufficiently interesting to warrant attention where increased delay is required together with lower impedance, as is clear from Table 4.

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MULTIFORM TOTAL SYMMETRY AND PARITY FUNCTIONS*

Indexing term: Switching terms

In recent letters, Biswas reported the existence of a special class of totally symmetric switching functions which remain invariant for all combinations of complemented and uncomplemented variables of symmetry, and also studied some of their properties. It is shown in this letter that these special symmetric functions are a particular class of linear functions known in literature as odd-parity functions and even-parity functions and form a subclass of totally symmetric functions showing multiform symmetry. Some of the relevant properties of these functions from the viewpoint of linear functions are also studied.

The properties of symmetric switching functions were studied by Shannon,[1] Caldwell,[2] McCluskey,[3] Marcus[4] and subsequently by many others. In a recent letter,[5] Biswas reported the existence of a special class of totally symmetric functions which remain invariant for all combinations of complemented and uncomplemented variables of symmetry. Subsequently,[6] the author presented a formal proof substantiating the existence of such symmetric functions and observed that, in every $n$cube, there are only two such symmetric functions F_o and F_e, which are compliments of each other. The object of this letter is to show first that these special symmetric functions F_o and F_e reported by Biswas are, in fact, a particular class of linear functions known in literature as odd-parity functions and even-parity functions, respectively, and belong to the subclass of totally symmetric functions showing multiform symmetry.[7] Some of the relevant properties of F_o and F_e from the viewpoint of linear functions are also discussed. It is shown that most of the results of Biswas follow quite readily by recognising these functions as linear functions. In the following we shall follow the notations and terminologies of Biswas as far as practicable. Some new terms which will be useful are introduced below.

Definition 1: A switching function of n variables $(x_1 x_2 \ldots x_n)$ can be considered as a dependent variable, depending on a subset of m variables, $0 \leqslant m \leqslant n$. When $m = 0$, the function is either 0 or 1, and is called a constant function. When $m = 1$, the function is either x_i or $\bar{x}_i$, $i = 1, 2, \ldots, n$, and is

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