

RANDOM NOISE MEASUREMENT WITH THE SPECTRUM ANALYZER

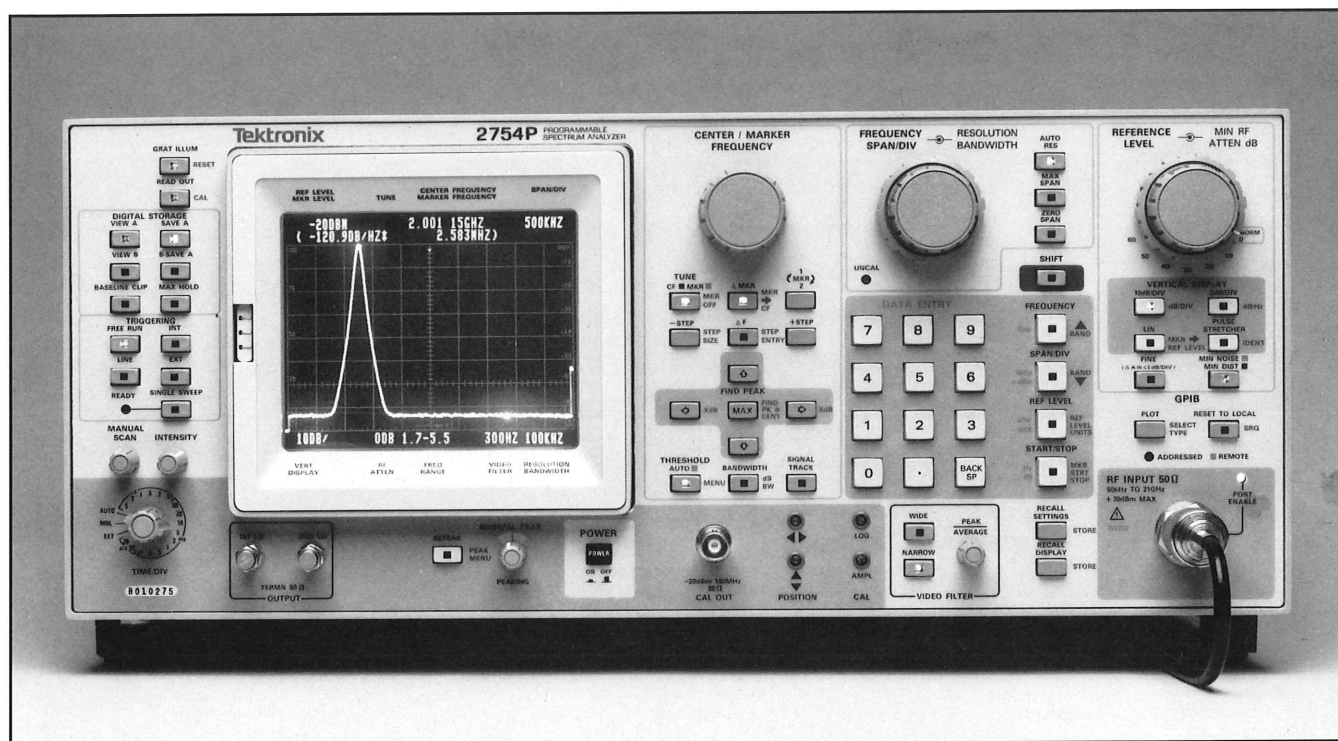


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Introduction

The need for noise measurement is as universal as the existence of noise itself. Whether the testing involves a signal-to-noise measurement, white noise loading in communications systems, oscillator spectral purity, or any other noise measurement, the basic need is the same: to measure noise power or noise power distribution as a function of frequency.

The spectrum analyzer is well suited to these measurements, provided appropriate procedures are followed. Described here are the details on how to correctly apply the spectrum analyzer to noise measurements. Most of the information is applicable to any spectrum analyzer, though some illustrative examples are instrument specific.

This paper treats the subject in five sections. The first deals with the nature of random noise. The second section discusses spectrum analyzer characteristics that affect the measurement. The third section concentrates on matters respecting random noise bandwidth. Routine, or common measurements are discussed in section four. Less common, or more difficult measurements are considered in section five. The paper concludes with a summary for quick review and reference.

1. Nature Of Random Noise

Random noise signals consist of the superposition of randomly occurring bursts, or fluctuations. These are usually due to natural phenomena, such as thermal agitation of electrons in a resistor. Hence the common name *thermal noise*. However, man made signals of a random, or more properly pseudo-random nature, such as digital radio modulation, also fit the description. The amplitude and phase of the random noise signal is completely irregular. This applies to both time and frequency. A granularity examination in time with an oscilloscope, and in frequency with a spectrum analyzer, will show similar displays. An important consequence of combining a large number of separate, randomly occurring fluctuations, is that these combine in accordance with the central limit theorem. This theorem states that a very large number of randomly occurring independent contributions to a common result

combine to form a normal, or Gaussian, probability distribution. Therefore, even though individual fluctuations are random, the average amplitude, or energy frequency distribution, i.e. power spectrum, are definable and measurable.

The random noise signal exhibits various interesting properties. For instance, the average sine and cosine components must be equal at every frequency in order to maintain a random phase distribution. The most important attributes, however, are that random noise is a power spectral intensity distribution measured per unit bandwidth e.g. watts/Hz, and that quadratic components (i.e. power) are additive. The basic measurement is in terms of watts/Hz, dBm/Hz, volts per root Hz, or some other variation along these lines.

Most naturally occurring random noise exhibits a uniform power vs. frequency spectrum, commonly called *white noise*. This natural tendency, to exhibit a very wide, constant amplitude spectrum, tends to be destroyed as the noise signal passes through various circuits such as filters or amplifiers. However, the basic characteristics of random noise, such as per unit bandwidth measurement, remain unchanged. Also, while the word, *noise*, conjures up an image of chaos and no information, in reality a random noise like spectrum can be an information bearing signal (e.g. digital modulation), and the statistical ensemble of the large number of random individual contributions is highly repeatable. These repeatable and predictable aspects of the random noise signal can be measured with a spectrum analyzer.

2. Spectrum Analyzer Characteristics

A spectrum analyzer is a specialized, calibrated, superheterodyne receiver. Such receivers operate by combining the incoming signal frequency with a local oscillator to produce an intermediate frequency (IF) where the signal characteristics are processed, or measured. Such receivers can exhibit spurious responses as various combinations of signal and LO frequency combine to the IF. This can result in incorrect identification of true signal frequency, though usually careful analysis will yield a true signal response. The problem is

more serious for random noise signals, especially those of wide spectrum width. The same input, converted to IF through multiple channels, will combine in accordance with properties of random noise to show a false result. If the image and true response combine together, the result will show twice the correct power level for a 3dB error. These sort of errors require quite subtle analysis to catch and sort out. It is a specialized topic that will not be discussed in this note. It is assumed here that the spectrum analyzer is free of significant spurious responses. This means, for instance, that microwave units are equipped with a spurious rejecting preselector.

The spectrum analyzer purports to measure the RMS, power, level of a signal. The amplitude level is usually calibrated in dBm, RMS. However, this is obtained by dividing the peak by $\sqrt{2}$; the ratio of peak to RMS for a sinewave. The output is processed by an envelope detector, and not by a power meter. Therefore, the amplitude calibration is wrong for random noise. Furthermore, the peak signal level is not a good starting point for measurements, because the random nature of the signal means that the peak level is time dependent. A larger peak will eventually occur if we wait long enough. However, the average and RMS levels are stable and have a fixed relationship to each other. What then is the correction factor, or ratio, of RMS to average value for random noise?

The spectrum analyzer envelope detector responds to the peak level of the random noise signal envelope. This is the same as the average value if the signal is averaged prior to detection. Also, the signal is amplified in the IF resolution filter chain prior to detection, and this limits its detected frequency bandwidth. Hence, the signal of interest is the average and RMS level of the envelope of band limited random noise. This is described by the mathematics of the circular, also known as Rayleigh, distribution. Unlike the symmetrical normal distribution, the circular distribution is not symmetrical. One can move only so far from any position towards the center of a circle, but there is no limit to how far one can go in the opposite direction. Though the probability of moving further out diminishes rapidly. This is equivalent to saying that there is no absolute limit to the peak amplitude of the envelope of band limited random noise. The probability of getting very large levels though, is very small.

The Rayleigh distribution is described by the probability density function

$$P(R) = \frac{R}{\sigma^2} e^{-\frac{R^2}{2\sigma^2}}$$

The standard deviation, σ (sigma), represents the RMS deviation from the average. The average value is

$$\sigma\sqrt{\frac{\pi}{2}}$$

and the RMS value of the envelope is

$$\sigma\sqrt{2}$$

The ratio of the two is

$$\sqrt{\frac{4}{\pi}}$$

or, 1.05dB at 20Log ratio. In other words, the RMS value we would like to obtain, is 1.05dB greater than the measured average value. An additional error develops when the measurement is made in a logarithmic vertical mode. Here the signal is compressed logarithmically before it is averaged, and the average of the log is not the same as the log of the average. The logarithmic function reduces the apparent contribution from the larger random noise perturbations so as to increase the display dynamic range. This effect has been computed at 1.45dB. Therefore, the total error in the logarithmic mode is 1.05+1.45=2.5dB.

Another spectrum analyzer characteristic that needs to be considered relates to internal spectrum analyzer noise, stated as a sensitivity or noise figure specification. Random noise power is additive. This means that the spectrum analyzer measures both the incoming noise signal, and the internal noise combined. Implications are discussed in the measurement section.

The final spectrum analyzer consideration is that of random noise bandwidth, B_n . It is necessary to know the measurement bandwidth since we are working in a per unit bandwidth mode. This is addressed in the next section.

3. Random Noise Bandwidth

Effect On Accuracy. Random noise is usually characterized in units of power per occupied frequency width. A random noise source that measures one milliwatt may in fact be the more powerful than one that measures one watt, if the first is measured in a one hertz bandwidth while the second was checked with

a one megahertz bandwidth. The effective random noise bandwidth is the average frequency width of the power, or voltage squared, filter response curve, i.e. the bandwidth of an equal area and amplitude rectangle. Unfortunately, neither the 6dB resolution bandwidth specified by Tektronix, nor the 3dB bandwidth used by some others is the same as the noise bandwidth. Furthermore, resolution bandwidth is specified at a poor, usually $\pm 20\%$ accuracy. Therefore, accurate measurements require that the effective random noise bandwidth of the spectrum analyzer be determined.

The user has various options in how to proceed, with variations in difficulty and accuracy. The situation also depends on the capabilities of the spectrum analyzer in use. The simplest, but also least accurate, option is to use the specified resolution bandwidth as if it were the random noise bandwidth. This will typically introduce a measurement error of up to 50%, i.e. 1.8dB. This error is in addition to all the other amplitude measurement errors, which are not related to the random noise nature of the signal, and are therefore not discussed in this note. The next step is to measure the actual resolution bandwidth, and thus eliminate most of the possibly 20% specification error. Some spectrum analyzers, including most Tektronix units, include an internal bandwidth measurement algorithm. The measurement is easy then. Otherwise it's more difficult, but still only a matter of a few minutes effort.

The next step in order of increasing accuracy, is to apply a correction factor. This correction can be applied to the specified bandwidth, or to the measured bandwidth for greater accuracy. A 10% to 20% additional accuracy improvement can be achieved in this way. The remaining measurement error due to bandwidth will thus drop to the 10% level. Correction factor multipliers come from experimental results or theoretical calculations. Martin provides the relationships between ordinary bandwidths, such as 3dB, and effective random noise bandwidths for cascades of various filter types.

Ultimate accuracy is achieved by measuring the random noise bandwidth of each filter of interest. Filter bandwidths can be measured directly by determining the transfer function response, and establishing the area under the power, (voltage squared), curve. An indirect method is to use

a random noise signal of accurately known watts/Hz level, so that the desired bandwidth can be computed by comparing measured amplitude against a CW standard.

Fortunately, these techniques are no longer necessary with many of the latest spectrum analyzers. Most new Tektronix units provide a dB/Hz function. Actuating this function lets the spectrum analyzer know that you are measuring random noise, and would like to have the result in dBm/Hz for a direct noise measurement, or dBc/Hz for a signal to noise measurement. The necessary 2.5dB correction, and noise bandwidth normalization are automatically accounted for from stored values in memory. Another reason for not pushing noise bandwidth determination beyond a certain point is the questionable return on effort. A 2.4% bandwidth error contributes just 0.1dB of amplitude error. Even the calibrator signal of the spectrum analyzer has a greater uncertainty with a ± 0.3 dB specification. Eliminating a 20% bandwidth error is worth an effort. A 2% error removal will be much more difficult, and provides only marginally improved accuracy.

Theoretical Computation. Random noise bandwidth correction factors are based either on experimental or historical results, or theoretical filter shape calculations. The first usually comes from the manufacturer. The second can come from the manufacturer, or be computed by the user. Some correction factor numbers floating about in the literature are suspect, especially if based on old measurements. It takes a stable unit to provide an accurate measurement. Older spectrum analyzers, lacking phaselock stabilization, are not good candidates for characterizing narrow filters. A good example is the 1.2X multiplier found in much of the literature. The beginnings of this number is about 20 years old. Current literature repeats statements from older literature that the random noise bandwidth for a gaussian filter is 1.2 times the 3dB bandwidth. However, the theoretically computed multiplier for a gaussian shaped filter is only 1.064. Furthermore, spectrum analyzers do not use gaussian shaped filters. Current products have either four section or six section synchronously tuned filters, for which the theoretical multipliers to the 3dB bandwidth is 1.128 and 1.105 respectively.

Theoretical, computed, relationships involving 6dB (B6), 3dB (B3), and random noise bandwidth (Bn) for various filter shapes is provided below.

Synchronously tuned number of sections	B6/B3	Bn/B3	Bn/B6
1	1.732	1.571	0.907
2	1.554	1.220	0.785
4	1.480	1.128	0.762
6	1.457	1.105	0.758
10	1.439	1.087	0.755
infinite (gaussian)	1.414	1.064	0.753
2 maximally flat 2-uples per CISPR EMI spec.	1.247	1.037	0.832

Direct Measurement. A very accurate direct measurement of bandwidth shape is difficult due to errors in determining frequency differences. The typical span linearity of 5% sets an upper limit on what can be achieved. To achieve greater accuracy it is necessary to use a signal source providing accurate frequency difference capability, and to measure display frequency differences against the external signal generator setting rather than spectrum analyzer span. There are spectrum analyzers that provide very accurate signal difference capability by use of "intelligent" markers that access the signal spectrum in digital storage location, rather than CRT display position, thus eliminating display linearity errors. However, such instruments also include a dBm/Hz function, so there is no need to measure the bandwidth in the first place. The procedure for determining random noise bandwidth from filter shape is illustrated as follows.

The effective random noise bandwidth is equal to the average frequency width of the power, i.e. voltage squared, response curve. Therefore, to determine this bandwidth one just needs to divide the area under the power curve by the height. The power response curve can be obtained in many ways. For example, measuring the linear, i.e. voltage, response and plotting the squares of the values is

one way. The example that follows involves a measurement in the logarithmic vertical mode with power response positions computed from the standard $\text{dBc} = 10 \log P$. It is clear from this relationship that a 20dB change represents a 100:1 power ratio. That is about as much as can be plotted on a reasonably sized graph paper. Another 10dB yields a 1000:1 ratio, requiring a significant increase in graph size. Impact on accuracy of ignoring the response below 20dBc is discussed later.

Figure 1 shows the top of the response curve in its most accurate display mode. The vertical display has been set at a sensitive 1dB/division (lower left readout), and the span is set to 15KHz/division (via keyboard entry) to display as much of the response as possible. The bandwidth 1dB down appears to occupy 3 divisions, or 45kHz. The more accurate value, measured with the marker, is 42.2kHz, for a 6.6% difference. After recording various bandwidth values, the vertical scale is set to 3dB/division, and span to

The next step is to transform the bandwidth measurements into a power shape relationship. Measurement results are shown below.

dBc	0.5	1.0	1.5	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20
RTO	.89	.79	.71	.63	.52	.50	.40	.32	.25	.16	.10	.063	.040	.025	.016	.010
kHz	30.3	42.2	52.8	60.5	67.7	73.4	84.4	94.2	103.4	120.8	136.7	152.8	168.4	183.7	199.2	215.4

20kHz/division to get the most accurate readings down to the desired 20dBc (fig 2). Correlation between the two displays was checked at 6dB. The delta marker setting shows an identical 103.4kHz in both cases, so the results correlate. Unfortunately for the reader, many spectrum analyzers will not permit peculiar settings such as 3dB, or 15kHz. Nor do all products provide accurate frequency markers. Therefore, the reader will have to settle for whatever settings, and consequent accuracy that can be obtained.

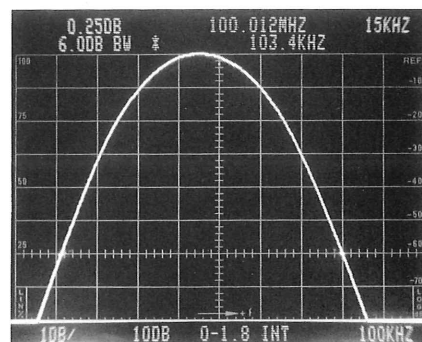


Figure 1. Determining the resolution filter shape.

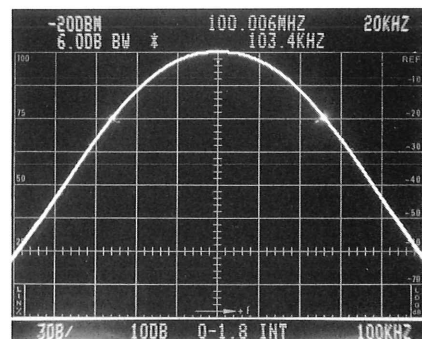


Figure 2. Determining the resolution filter shape.

You now need to plot the power response curve on calibrated graph paper. In this example, the numbers were plotted on a B size graph with 25 vertical divisions representing 0dBc, or unity ratio, and a horizontal setting of 10kHz per division. Thus, at 4dBc, and a power ratio of 0.4, we have 10 vertical divisions for a 84.4kHz bandwidth. At 10dBc the vertical display is down to 0.1, or 2.5 compared to a full height of 25 boxes. And so forth. The area under this curve is now found by counting boxes. It turned out to be 196 boxes, for an average width of $196/25=7.84$. This is a 78.4kHz effective random noise bandwidth at 10kHz/box. In other words, an equal height, (25 boxes), rectangle 78.4kHz wide equals the power curve in area. The ratio to the 6dB bandwidth is $78.4/103.4=0.758$.

The random noise bandwidth measured/computed in this example is less than the true value because the area below 20dBc has been ignored. The amount of error can be determined by comparing the area under the curve to infinity versus the limit yielding 20dBc. For example: 20dBc is 0.1, and the response for two synchronous stages in cascade is

$$\left(\frac{1}{\sqrt{1+x^2}} \right)^2$$

Therefore, $x=3$. The area under the power, voltage squared, curve is the integral of

$$\frac{dx}{(1+x^2)^2}$$

which is equal to

$$\frac{x}{2(1+x^2)} + \frac{1}{2} \tan^{-1} x = 0.775$$

The result to infinity is $\frac{\pi}{4}$ or 0.785.

The error is $0.785/0.775=1.013$. Error values for synchronously tuned stages is as follows

Nr. Stages	1	2	3	4	gaussian
% Error	6.8	1.3	0.8	0.6	0.3

The appropriate value for the four stage filter used in the example is therefore, $(0.758)(1.006)=0.763$, almost identical to the theoretical value of 0.762.

4. Basic Measurements

Peak/Average Ratio. Figure 3 shows the spectrum shape of a random noise signal. The upper display shows the peak detected envelope without any averaging. The level jumps about as noise spikes of different amplitude fluctuate in accordance with the probability of the Rayleigh distribution. The lower trace shows a steady average level, using video filter and digital averaging. The 10.4dB difference between the traces is typical for most spectrum analyzers. The averaged value should, of course, be the same for all spectrum analyzers set to the same random noise bandwidth. The displayed peak value, however, will vary depending on sweep time and peak-hold circuit characteristics. The longer the sweep time the more likely that a larger peak might occur. This peak will be displayed if the peak-hold circuit will respond to the narrow noise spike. Therefore, a larger ratio of peak to average noise display on a spectrum analyzer is not indicative of an inferior product, as some might think.

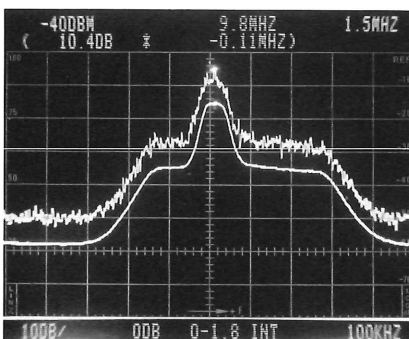


Figure 3. Random noise signal, peak/average difference.

The degree to which possible noise fluctuations are caught and displayed can be calculated, as follows.

The area under the probability function, found by integrating the Rayleigh distribution is:

$$1 - e^{-\frac{R^2}{2\sigma^2}}$$

This yields unity, indicative of a certain event, for levels to infinity. We know that the average value is 1.25σ , and the ratio is 3.3 for a 10.4dB peak/avg

difference. Hence the peak level is $(3.3)(1.25)=4.13$. Therefore, we are missing only

$$e^{-\frac{(4.13)^2}{2}} = 0.02\% \text{ from a certain}$$

event. In other words, we are capturing 99.98% of possible noise level occurrences.

Clearly, an accurate measurement requires a stable and correct display of average noise level. Sutcliff has shown that the error in average noise level display due to insufficient averaging is given by

$$\sqrt{\frac{2B_v}{B_n}}$$

For example, if the averaging video filter bandwidth is 1/100 that of the noise bandwidth, the error is

$$\sqrt{\frac{2}{100}} = 0.14.$$

The value measured with insufficient averaging will be larger than the actual average level. Figure 4 shows the effect of insufficient averaging. The upper trace used a 3kHz video bandwidth with a 100kHz resolution bandwidth (about 80kHz noise bandwidth). The lower trace is based on a high level of smoothing. The measured difference between the traces is 2.4dB. In fact this could be anywhere from near zero to over 5dB. The computed error is based on the average peak value, while the measured result depends on whatever level of noise fluctuation the marker happens to be on. The important point is to understand the need for noise smoothing. The usual practical rule is to run at a 300:1 or greater bandwidth ratio.

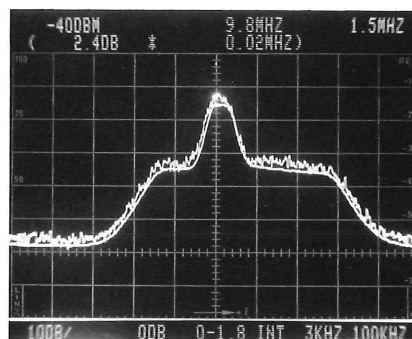


Figure 4. Illustrating insufficient averaging.

Determining dBc/Hz. The absolute amplitude level of the noise signal is measured at -54dBm in figure 5. This measurement is based on a $100\text{kHz} \pm 20\%$ resolution filter setting. Hence the actual 6dB bandwidth can be anywhere from 80 to 120kHz , and the random noise bandwidth is between 61 and 91.4kHz , assuming a 0.762 multiplier. A full measurement would require a determination of the bandwidth, computation of the dB difference to 1Hz , and addition of the 2.5dB correction. With the dB/Hz function of the 2756P , we get the result directly at -101.3dBm/Hz shown in figure 6. Note that the video filter bandwidth is set at 300Hz compared to 100kHz for the resolution. Also the peak/average cursor is up so that the spectrum includes digital averaging. More averaging will show a smoother display, but it takes longer to do the measurement. Most experienced operators do not try to get a particular smoothing bandwidth. Video bandwidth and/or digital averaging, is set for an acceptably smooth display in compromise with measurement time.

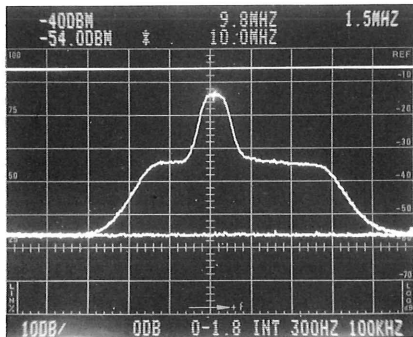


Figure 5. Noise spectrum determination, no normalization or error correction.

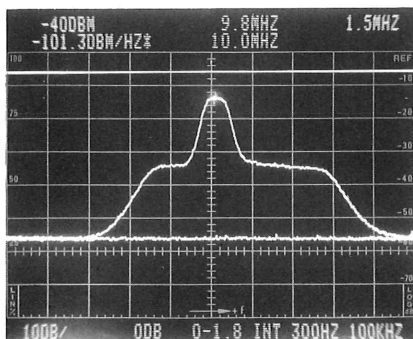


Figure 6. Noise spectrum determination with bandwidth normalization and error correction.

It is obvious by observation that we are not dealing with constant amplitude, white noise. The frequency width at the base of the upper signal portion shows as 2.13MHz in figure 7. This measurement was made using the bandwidth function of the spectrum analyzer. A less "intelligent" unit would require a bit more user effort to get the information. Note that the bandwidth function also shows the relative amplitude level as 19.2dB . The next lower spectrum width is 6.51MHz at 22.8dBc , per the marker setting in figure 8.

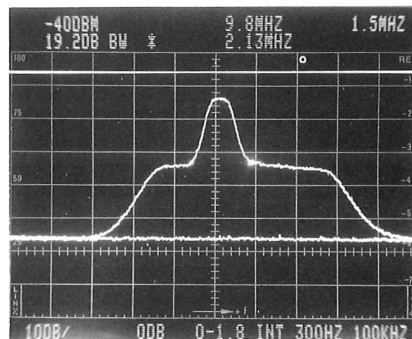


Figure 7. Random noise signal spectrum shape determination.

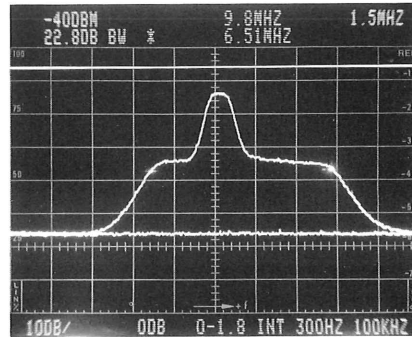


Figure 8. Random noise signal spectrum shape determination.

Choice Of Bandwidth. Additional, lower level, signal spectrum features can not be observed as the internal spectrum analyzer noise (lower, straight line trace) gets in the way. At 10dB/div , the display dynamic range appears to be about 40dB . A more accurate measurement, using delta markers, shows 41.2dB in figure 9. The usual procedure, when encountering insufficient signal display range, is to reduce the resolution bandwidth. Lower bandwidth cuts internal noise, improves sensitivity, and increases dynamic range. But, this will not work when the signal is random noise. The signal to internal noise ratio remains unchanged with changing measurement bandwidth, as both are in units of dBm/Hz . The only way to increase the display dynamic range is to increase the input signal level, or use a low noise preamplifier. This procedure can also cause problems, such as mixer overload or intermodulation, as discussed in the advanced measurements section.

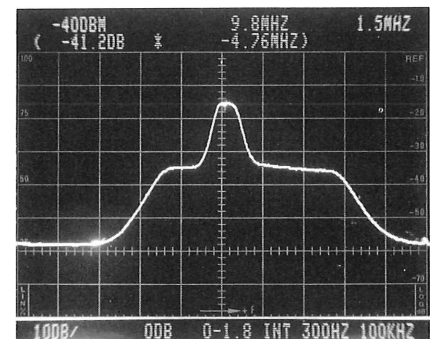


Figure 9. Signal noise to spectrum analyzer internal noise level difference determination.

Though the dynamic range, or signal to noise ratio does not change with changing bandwidth, the ability to make accurate measurements does. Figure 10 shows two traces. The upper used a 100kHz bandwidth as in the previous displays. The lower trace used a 10kHz bandwidth. Spectrum shape and display dynamic range remain unchanged. Signal amplitude is measured as -101.4dBm/Hz , within normal measurement accuracy/repeatability of the previously obtained -101.3dBm/Hz . Going the other way, however, causes errors. Figure 11 shows a 1MHz based trace in comparison with one taken at 100kHz. The signal spectrum is broader with rounded corners. The upper level width measures 2.89MHz, rather than 2.13MHz previously measured. The generally accepted rule is that the measuring bandwidth to spectrum width should have at least a 3:1 ratio. No measurable difference is observed beyond a 10:1 ratio. A roughly 2MHz wide feature exhibits noticeable shape distortion when measured with a 1MHz bandwidth. A 700kHz bandwidth would be barely adequate.

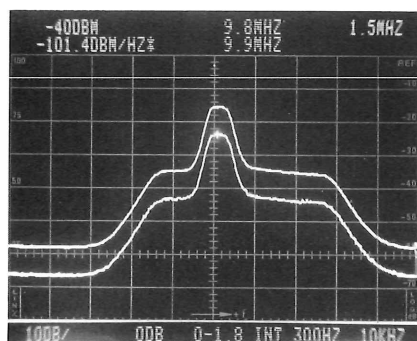


Figure 10. Effect of resolution bandwidth on spectrum shape determination.

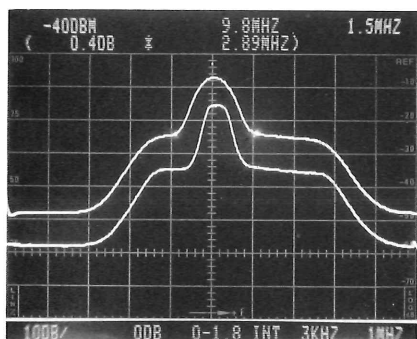


Figure 11. Effect of resolution bandwidth on spectrum shape determination.

Close observation of figure 10 shows what appears to be a CW signal about 4.8 display divisions to right of center. This signal became clearly discernible at the lower, 10kHz, resolution bandwidth. Unlike the noise signal, we do get improved signal to noise ratio at a narrower bandwidth. We can center this CW signal by tuning the center frequency control, by direct entry of a computed approximate frequency, $9.9 + (4.8)(1.5) = 17.1$, or marking the signal followed by a center command. Whatever the procedure, figure 12 shows the CW signal at 100Hz bandwidth for good signal to noise ratio. Amplitude level measures -99.6dBm , at 17.064472MHz measured with the frequency counter. These values can be compared to various noise signal features, such as -101.4dBm/Hz at 9.9MHz . CW to noise signal level is $-99.6 - (-101.4) = 1.8\text{dBc/Hz}$, for example.

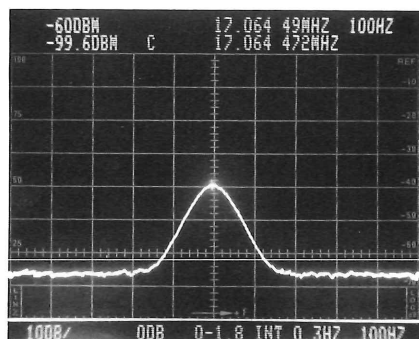


Figure 12. Pulling a CW signal out of the noise.

Correcting For Internal Noise. Suppose we are interested in the noise signal at some particular location in

the spectrum. We measure the spectrum level at that location at 8dB above spectrum analyzer internal noise, as illustrated in figure 13. This, relatively small, amplitude difference between the measured noise and internal noise should alert the user to additional errors. Recall that noise powers are additive. The spectrum analyzer is actually measuring the sum of incoming and internal noise. Hence, the result is greater than the incoming noise level to be measured. Input noise equal to internal noise will show a 3dB error, since $1+1=2$, and $10\text{Log}2=3$. Some, rare spectrum analyzers, such as the Tektronix 2710, provide internal correction for this effect. Most instruments do nothing. The user is then obliged to make a manual correction if accuracy is to be maintained.

For the example shown in figure 13, the table indicates a 0.75dB correction. The correct value is then $8 - 0.75 = 7.25\text{dB}$.

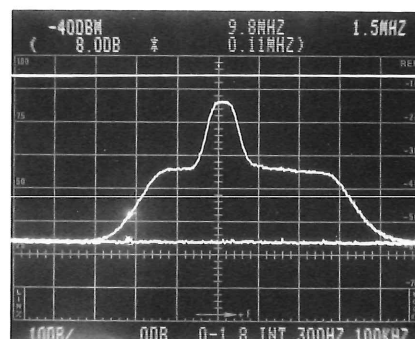


Figure 13. Incoming signal noise close in amplitude to internal noise.

The correction is easy to compute on the basis of two combining noise powers. The result can be graphed, or set up in a table as below.

Measured Noise, dB Above Internal	1.5	2.0	2.5	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Actual Noise, dB Above Internal	-3.9	-2.3	-1.1	0.0	1.80	3.35	4.74	6.03	7.25	8.42	9.54
Actual Noise, dB Below Measured	5.35	4.33	3.59	3.01	2.20	1.65	1.26	0.97	0.75	0.58	0.46

5. Advanced Measurements

Error Correction: The last discussed item deals with correction of errors due to internal noise. One currently (1989) available spectrum analyzer, the Tektronix 2710, makes this correction automatically for measured vs internal noise levels of 2 through 10dB differences. The dB/Hz algorithm not only takes care of the usual bandwidth and 2.5dB factors, but also the effect of internal noise. Internal noise values are stored in memory whenever a sensitivity calibration test is performed. These values are used in a correction factor to measured noise in accordance with the table on the previous page. This makes noise measurement easier using the 2710. However, the user needs to be aware that this correction is performed automatically, and not cause errors by adding a second, manual, correction. This automatic correction can also cause confusion, or errors when checking spectrum analyzer internal, self generated noise.

Another random noise measurement area where the Tektronix 2710 excels, deals with non-sinusoidal signals. The displayed amplitude level of non-sinusoidal signals, such as those used in broadcast television, will be significantly affected by averaging. But averaging must be used to measure random noise. Signal to noise determination for such signals must therefore, be performed as a sequence of steps. The signal and noise levels are obtained separately, and the dB difference is computed from the individual results. The 2710 does it all in one step. The spectrum analyzer centers the noise marker position, reduces span to zero, engages the video filter, and measures the resulting noise level whenever the noise dB/Hz, or signal to noise ratio dBc/Hz functions are used. This is all done during retrace so user convenience is not affected. The implications of the above is illustrated below.

Figure 14 shows a noise signal (upper trace) at -86.0dBm , about 40dB above 2710 spectrum analyzer internal noise (lower trace). Figure 15 shows the same display in the dBm/Hz mode at -129.3dBm/Hz . Figure 16 shows the same random noise signal without any averaging. The peak display level is about one division, (10dB @ 10dB/div), above the video filter averaged display of

figure 15. However, the normalized noise level shows the same -129.3dBm/Hz , because it is measured under averaged conditions during retrace. Figure 17 attempts to measure the 2710's self generated noise. The normalized level shows as -173.6dBm/Hz . This represents a 0.4dB noise figure for the internal preamplifier (see PRE, upper left), since an ideal, 0dB, unit is at -174dBm/Hz . The 2710 preamplifier is low noise, but not that spectacular. The clue to what is going on is in the error message, "noise level less than 2dB". The 2710 corrects for measured noise down to within 2dB of internal noise. Naturally, the internal noise is not 2dB greater than itself. The 2710 applies a correction factor, yielding a much lower apparent noise level than the true level. In this case the error is fairly obvious. A 0.4dB noise figure is suspicious. Not so the case for figure 18, with the preamplifier turned off. The noise figure appears to be very good for an RF spectrum analyzer, but not out of the realm of reason. That is why an error message is provided.

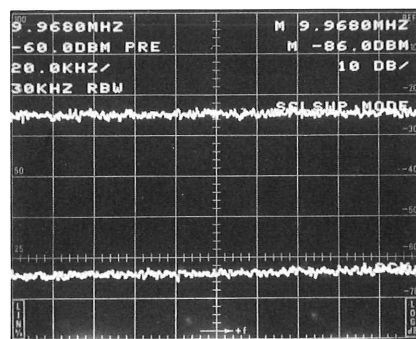


Figure 14. Incoming signal noise is well above internal noise.

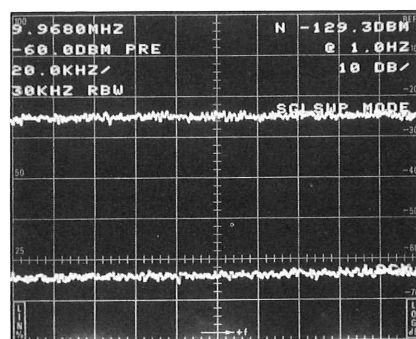


Figure 15. Same as 14 but in dBm/Hz mode, using normal video filter averaging.

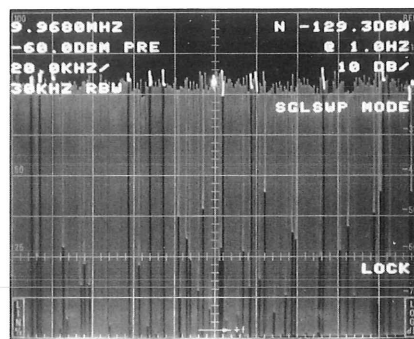


Figure 16. Same as 15 but using retrace mode averaging.

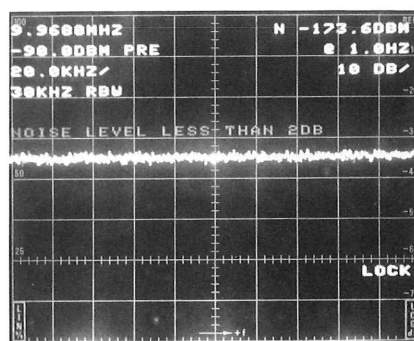


Figure 17. Correction for internal noise when measuring the same internal noise introduces error message.

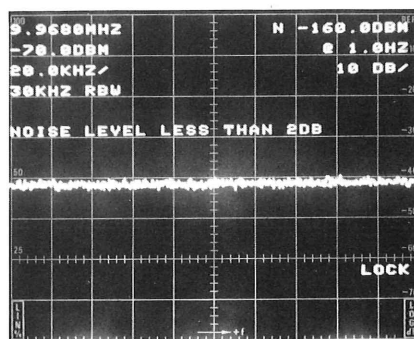


Figure 18. Correction for internal noise when measuring the same internal noise introduces error message.

The relative noise level self correction feature comes in handy when dealing with low noise levels, as illustrated in figure 19. The measured noise (upper trace) shows as -109dBm , or 3.4dB above the internal noise measured in figure 20 at -112.4dBm . When the large noise level was measured, the correction plus normalization value was found to be $129.3 - 86 = 43.3$, (see figs 14&15). The low noise level shows a normalized value of -155.3dBm/Hz , figure 21. This is a $155.3 - 109 = 46.3$ normalization value. The two are in good agreement at 3.4dB versus $46.3 - 43.3 = 3\text{dB}$. The procedures described for noise alone measurement are equally applicable to noise in signal to noise, also known as carrier to noise, measurement. Figure 22 shows a two marker display, one on the carrier and the other on the noise. The noise is normalized per hertz, the 2.5dB correction is added, and effect of internal noise is accounted for. The result is compared to the carrier level determined with the other marker. Figure 22 shows a measured value of 53.3dB/Hz amplitude difference.

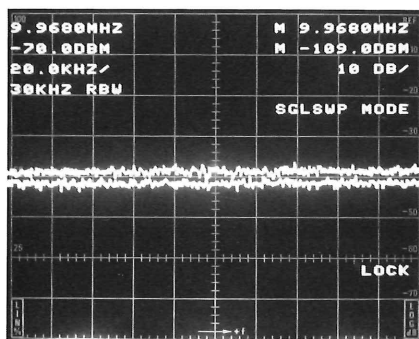


Figure 19. Incoming noise measures -109dBm .

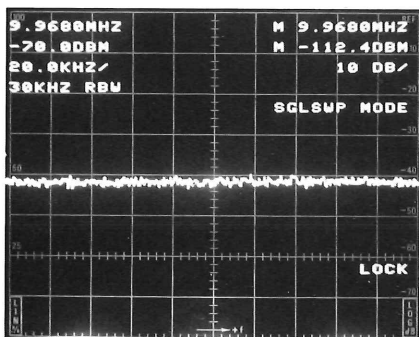


Figure 20. Internal noise at -112.4dBm is 3.4 dB below fig 19.

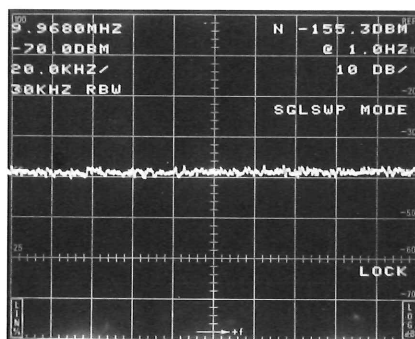


Figure 21. Same as 19 but normalized to dBm/Hz , and corrected for internal noise.

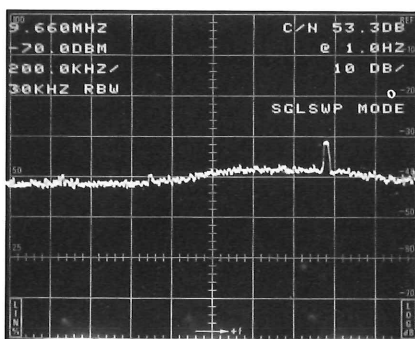


Figure 22. Carrier to noise measurement using markers.

Relative Level: It has been previously explained that random noise spectrum shape is not affected as long as the measuring bandwidth is sufficiently narrow (less than $1/3$ of spectrum width). This also means that relative amplitude measurement, within an amplitude varying noise spectrum, can be accurately performed at any, sufficiently narrow, resolution bandwidth setting. The absolute display level in dBm will change as the bandwidth is changed. But, normalized amplitude in dBm/Hz is not affected, and neither is the dB difference between spectrum features. This was demonstrated earlier in connection with figure 10. Another consequence of this reasoning is that one can compute, or normalize, the noise level with respect to any chosen bandwidth. There is nothing special about 1Hz , except that it is an easy number to work with. Thus, the -101.3dBm/Hz value of figure 6, becomes -71.3dBm/kHz with the addition of $10\text{Log}1000 = 30\text{dB}$. For a 4kHz channel we add $10\text{Log}4000 = 36\text{dB}$, etc. A calculation using simple bandwidth scaling will yield accurate results provided the noise amplitude

is relatively flat within the bandwidth of interest. Otherwise converting from a spot frequency value to a broadband value will be in error. The 2.13MHz width measured in figure 7, indicates that a normalization to a 4kHz audio channel bandwidth is fine. Normalization to a 4MHz video channel bandwidth will be in error. What to do about this is discussed later.

An important aspect of the above is how to determine amplitude level with respect to any arbitrary normalization bandwidth. A case in point involves digital radio. The digital radio signal though man made, and not representative of a natural phenomenon like thermal noise, behaves in all respects like random noise. The Federal Communications Commission (FCC) specifies the maximum limits of spectrum bandwidth occupancy. This specification is usually represented by a graph known as the FCC mask. Figure 23 shows the basic shape of this "mask". The basic shape of the mask is universal, though requirements may vary depending on the radio. For a 6GHz radio, for instance, the maximum 50dB down bandwidth at the throat of the mask is 30MHz . The specification further states that, the 50dB spectrum width is to be determined for a 4kHz channel bandwidth, and with respect to the "mean output power level". Does this mean that we have to make the measurement with a 4kHz resolution bandwidth, or maybe it should be a 4kHz random noise bandwidth? The 4kHz noise rather than resolution is correct. But we only have to normalize to that bandwidth, rather than measure with it. Furthermore, as far as spectrum shape is concerned, it makes no difference what bandwidth is used as long as it's narrow compared to the spectrum width. With a 30MHz spectrum width, a 4kHz bandwidth or a 400kHz bandwidth are all the same.

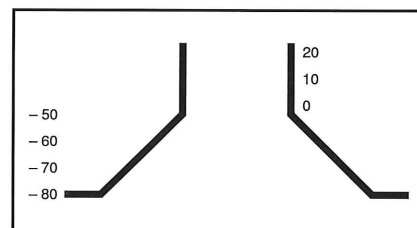


Figure 23. FCC mask for digital radio transmission spectrum.

The real problem is to know from where to measure. What is the "mean output power level?" If the radio is out of service, then we can find the mean power by checking the unmodulated carrier. A simple alternative is to follow manufacturer specifications respecting a comparison of modulated spectrum level and mean output power. Finally, the user can compute the value based on modulation relationships. The implications of such a calculation, 10Log (measuring BW/modulation signaling rate), is shown in figure 24. A typical signaling rate of just over 25MHz yields a 38dB mean power to spectrum level difference. This means that we would measure the 50dB bandwidth $50-38=12\text{dB}$ down from the top of the spectrum. The measuring bandwidth is not of any importance here. The same logic also applies to the 80dB sidelobe suppression requirement called out by the mask. The measurement would check for at least $80-38=42\text{dB}$ suppression with respect to spectrum peak.

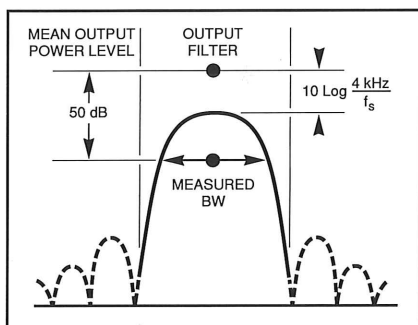


Figure 24. Digital radio spectrum output level relative to mean output power level.

All the previous measurements are of a relative nature. We are determining power levels, or occupied spectrum width, with respect to mean transmitted power level. A measurement of what that mean power is however, requires knowledge of the actual random noise bandwidth used.

Thus, if the peak spectrum level happens to measure -53dBm in a 100kHz resolution bandwidth, and this happens to be -100dBm/Hz , either from direct spectrum analyzer measurement or manual computation (e.g. we find the noise bandwidth @ 89.1kHz for $-10\text{Log}89100+2.5-53=-100$), then: for a 25MHz signaling rate the mean power is $-100+10\text{Log}25\text{MHz}=-26\text{dBm}$. Likewise, $-53+10\text{Log}(25\text{MHz}/89.1\text{kHz})+2.5=-26\text{dBm}$. Adding to this, the pickoff coupler or attenuator dB ratio will yield the full radio output level.

Total Power. The random noise signal is measured in units of power per occupied spectrum width. The spectrum analyzer only measures at a chosen frequency spot. To find total power for white noise, where amplitude is constant across the occupied bandwidth, is just a matter of multiplying by the ratio of occupied to measured bandwidth. In effect, we normalize the measurement against occupied bandwidth. Most random noise signals however, do not have a flat spectrum. The simple expedient of multiplying by bandwidth ratios, or adding a dB equivalent, will not work. Some spectra have well defined shapes. The relationship of area under the spectrum shape to individual levels, such as the peak level, is mathematically defined. Such was the case for the digital radio spectrum example, where the total power was related to per hertz peak spectrum shape power. A mathematical computation will then yield the desired total power. For an arbitrary spectrum shape it is necessary to add power, or area under the spectrum shape, on a piece-wise basis as illustrated below.

Figures 5 through 9 show amplitude and frequency details for an arbitrary random noise spectrum shape. Figure 25 shows a width at the base of 10.94MHz , and figure 26 shows a pedestal amplitude of -74.4dBm . From this information, plus adjustment

for the effect of internal spectrum analyzer noise on the lower amplitude levels, we obtain the basic spectrum information shown in figure 27. We now compute and add the absolute power levels based on the areas under the spectrum shape. The primary difficulty is that the addition must be based on areas under the power (watts) spectrum shape, and not the logarithm of that shape (dBm). Fortunately, most shapes can be approximated by a combination of triangles and rectangles. The best shape is the rectangle because it remains invariant for vertical scale transformations, such as log-dBm to power-watts. Triangles are a bit more complicated, but not too difficult once the basic formula is known.

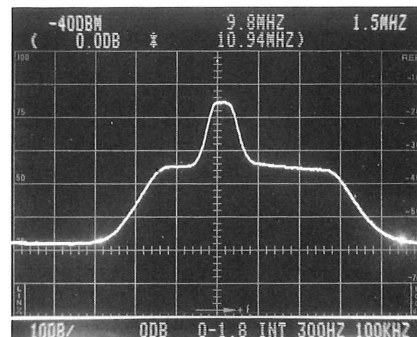


Figure 25. Additional spectrum shape information to that obtained in figs 5-9.

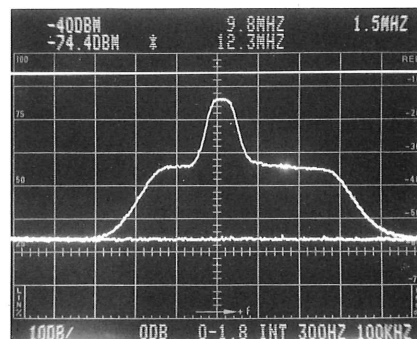


Figure 26. Additional spectrum shape information to that obtained in figs 5-9.

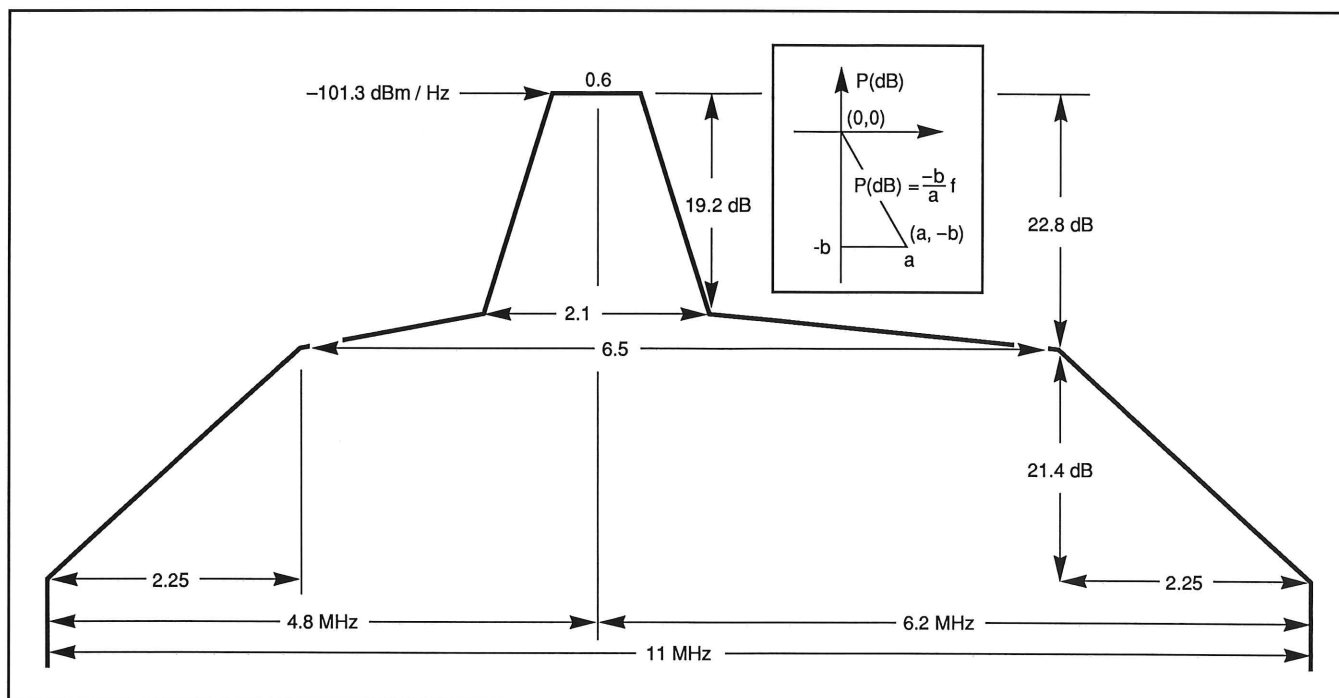


Figure 27. Spectrum shape details.

The intercept form of the straight line forming the hypotenuse of the triangle extending $-b(\text{dB})$ on the y (log-power) axis, and a (Hz) on the x (frequency) axis, is:

$$P(\text{dB}) = \frac{-b}{a} f \text{ (insert fig 27).}$$

From $P(\text{dB}) = 10\log P$ we have that

$$P(f) = 10^{-\frac{b}{10a}f}$$

We also know that

$$10^x = e^{\frac{x}{0.4343}}$$

Therefore,

$$P(f) = e^{-\frac{b}{4.343a}f}$$

(e is the base of natural logarithms, 2.718). Then the area under the power curve is

$$P(\text{total}) = \int_0^a e^{-\frac{b}{4.343a}f} df =$$

$$P(\text{total}) = \frac{4.343a}{b} \left[1 - e^{-\frac{b}{4.343}} \right]$$

This is not just the area of the log scale triangle, but the total extending to minus infinity dBm (i.e. zero power). Thus,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!},$$

$$@ b \rightarrow 0, e^{-\frac{b}{4.343}} = 1 - \frac{b}{4.343}$$

and $P(\text{total}) = a$.

As the triangle on top disappears, we are left with the area of the bottom rectangle, a (Hz) wide and amplitude normalized to unity.

Computing the values for figure 27. For the upper trapezoid, $b=19.2$ dB, and $a=0.75$ MHz. For one triangle:

$$\frac{(4.343)(0.75)}{19.2} \left[1 - e^{-\frac{19.2}{4.343}} \right] = 0.1676 \text{ MHz}$$

The two triangles of the upper trapezoid are equivalent to a 0.335 MHz rectangle, which together with the 0.6 MHz center rectangle, occupy 0.935 MHz. The total power is

$-101.3 \text{ dBm/Hz} + 10\log 0.935 \text{ MHz} = -41.6 \text{ dBm}$. This is the total power under the central 2.1 MHz. The next, 3.6 dB deep, trapezoid has a 2.1 MHz central rectangle, and two triangles 2.9 MHz and 1.5 MHz wide. The rectangle is already accounted for, so only the triangles remain. These compute to an equivalent width of 1.97 MHz and 1.02 MHz. The power contribution is $-101.3 - 19.2 + 10\log(1.97 + 1.02) \text{ MHz} = -55.7 \text{ dBm}$. Similarly, the lower trapezoid contributes -64.5 dBm .

We add the powers by setting the largest level to unity. Thus, if -41.6 dBm is one, then -55.7 dBm is 0.039, and -64.5 dBm is 0.005. The total of 1.044 is a 0.2 dB increase, for -41.4 dBm . The small increase could have been predicted by observation of the spectrum shape. Note that a 20 dB change is a 100:1 power ratio. Therefore, it takes 100 MHz lower down to contribute as much as 1 MHz on top. The additional 0.2 dB is hardly worth the effort, especially given the poor accuracy of the procedure.

However, for relatively flat and/or wideband noise the results can be quite different. Observation of a safe seeming level of -60dBm/Hz is really $+30\text{dBm}$, or 1 watt, for a 1GHz spectrum width. A greater input level will damage most spectrum analyzers resulting in costly repairs. Much lower total noise signal levels, while not damaging to the equipment, may still be too much as a result of measurement errors due to intermodulation. A $+5\text{dBm}$ intercept point, typical of most spectrum analyzers, will result in distortion products less than 30dB down for a -10dBm power level input. Fine spectrum shape features will be obscured. There is no easy way to recognize this, because by its basic nature all random noise looks the same. A check that display level changes one for one with input level is the easiest way to test for distortion products. A more quantitative test involves noise loading around a transmission channel, or through a notch filter. Any noise that appears within the transmission or notch filter bandwidth must be generated by distortion products.

Other Noise. Not all noise is random noise. Impulsive noise, caused by random, stray, or just undesired pulses is the dominant form of "noise" interference due to man-made devices. This is a major topic in its own right, of primary interest to the electro-magnetic interference, EMI, community. Impulsive noise is a voltage spectral intensity distribution, measured in units of volts/Hz, as opposed to watts/Hz for random noise. Measurement techniques, correction factors, bandwidth determination . . . are all different than for random noise. Measurement of impulsive noise is not discussed in this note. What is discussed, is how to recognize what type of noise you have.

Figure 28 shows a CW signal within noise display, all well above internal spectrum analyzer noise shown in the lower trace. The signal to noise

ratio, measured with markers and internal noise normalization algorithm, shows as -56.5dB/Hz . The assumption inherent to this measurement is that the noise is random noise. But what if it isn't? One way is to check that we really are dealing with a dBm/Hz type signal. Measuring the normalized noise level with a different resolution bandwidth will show whether we get the same dBm/Hz result, or not. An alternate method, that does not require a unit with advanced dB/Hz computation features is illustrated in figure 29. A change in bandwidth of ten times should result in a display amplitude change of 10dB . The bandwidth change from 100KHz to 10KHz shows an amplitude drop of 10.4dB . Most, if not all of this signal is random noise. A closer assessment would require a comparison of resolution filter random noise bandwidths, rather than specified 6dB bandwidths. Figure 30 shows the same measurement as in figure 29, on a slightly different signal. The amplitude difference is now 17.6dB , too much off from 10dB to be due to bandwidth specification tolerances. In fact, the signal in figure 30 is a combination of the figure 29 random noise signal and an impulse noise signal, shown in figure 31. An impulse noise signal should change by 20dB for a ten times bandwidth change. Figure 31 shows a difference of 21.2dB .

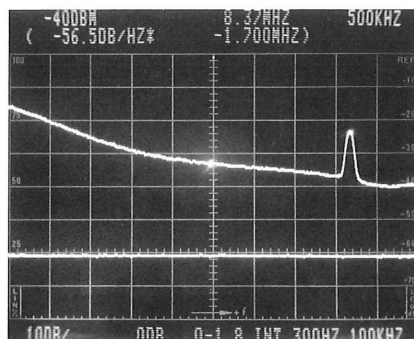


Figure 28. CW signal within noise spectrum.

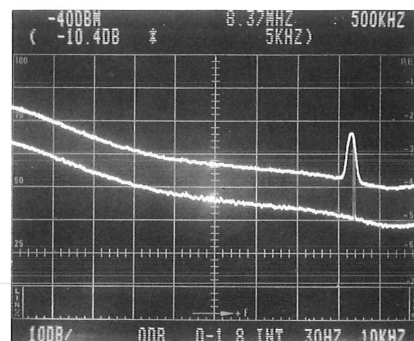


Figure 29. Check that the noise in 28 is random noise.

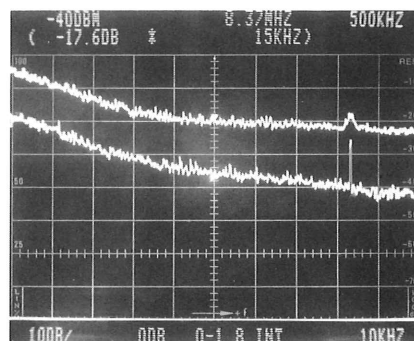


Figure 30. Similar to 29, but 17.6dB level change for $10\times$ bandwidth changes shows noise is not all random.

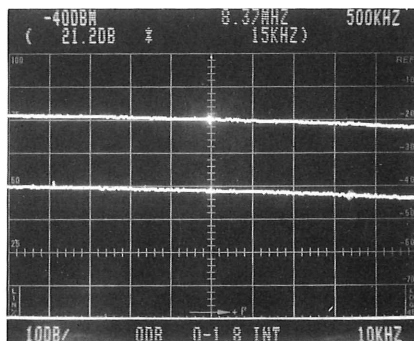


Figure 31. Impulse noise combined with 29 to yield 30.

A totally different method of recognizing the type of noise, and obtain other information as well, is to analyze the shape of the probability density function of the (presumed) random signal. Figure 32 shows a single scan of a random noise signal using linear, i.e. voltage, vertical mode. The display is not real time, or analog, but rather the digital storage version of the envelope detector output.

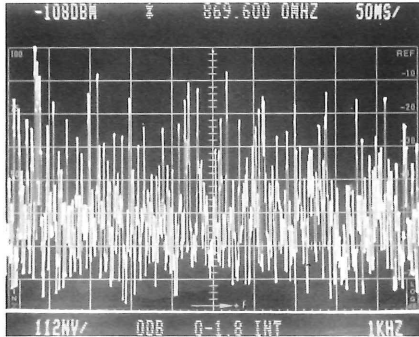


Figure 32. Random noise amplitudes in zero span voltage scale.

For this particular spectrum analyzer, the horizontal scale consists of 1000 points from the interlace of two 500 point memories. The vertical, voltage, scale is digitized into 256 discrete levels. We can choose one, or the other, memory for analysis, resulting in 500 independent signal level samples per sweep. With the help of an external computer, we can store and analyze as many samples as we wish at the rate of 500 per sweep. Two sweeps results in figure 33, read out of computer memory. The horizontal scale shows sample numbers from zero to 1000. The full vertical level is 256. The computer will now perform a sort and totalize operation, to determine how many samples fall within each of the vertical voltage levels. The result, normalized to an amplitude variation of zero to 100, is plotted on the histogram of figure 34. The peak at about the twenty percentile amplitude level shows 24 intercepts, the peak at the 45% point shows 18 intercepts, etc. We get some idea of the probability function, but 1000 points is not enough of a sample. Figures 35 and 36 show the result for ten sweeps, providing 5000 points. Here, figure 36 is clearly a crude representation of the Rayleigh distribution shown in figure 37. An analysis of samples from a hundred sweeps should provide a fine grain detail of the distribution.

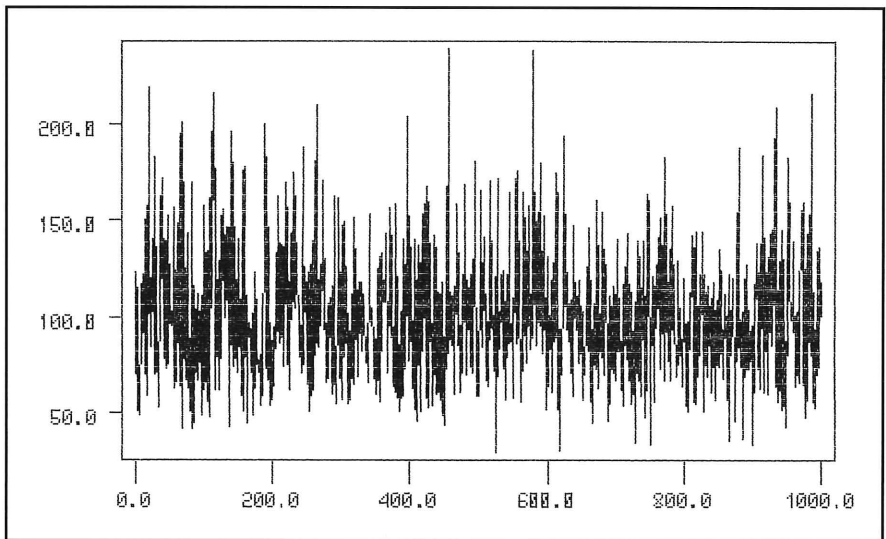


Figure 33. Two sweeps of 500 samples each combine for 1000 voltage level samples.

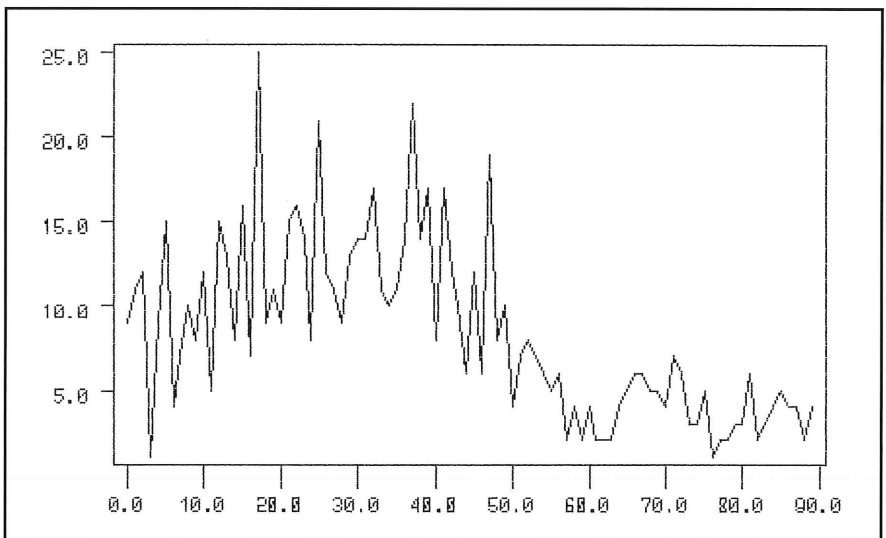


Figure 34. Probability distribution histogram from fig 33 samples.

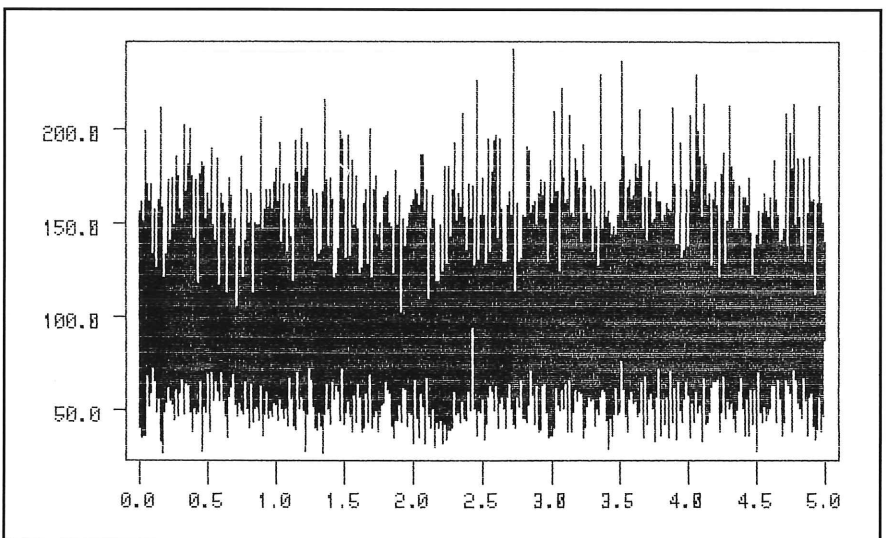


Figure 35. Same as 33 but 5000 sample points from 10 sweeps.

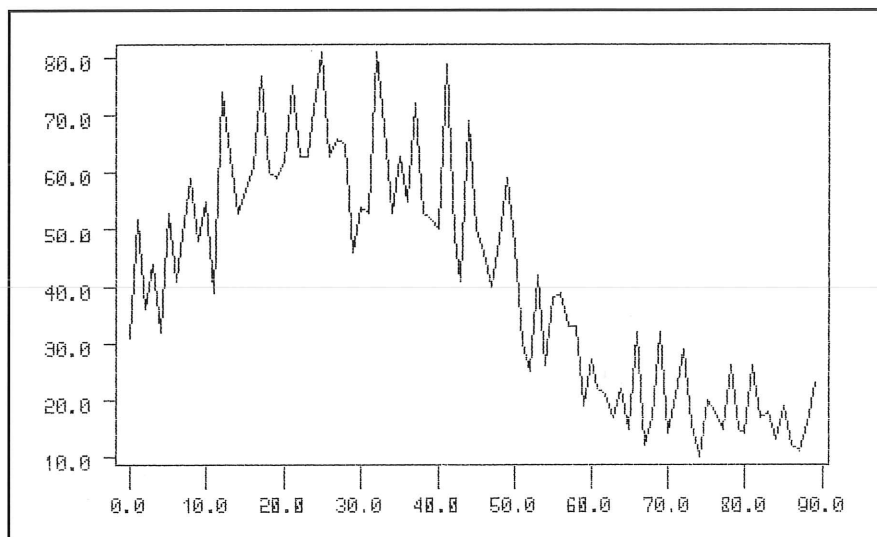


Figure 36. Same as 34 but based on 5000 samples.

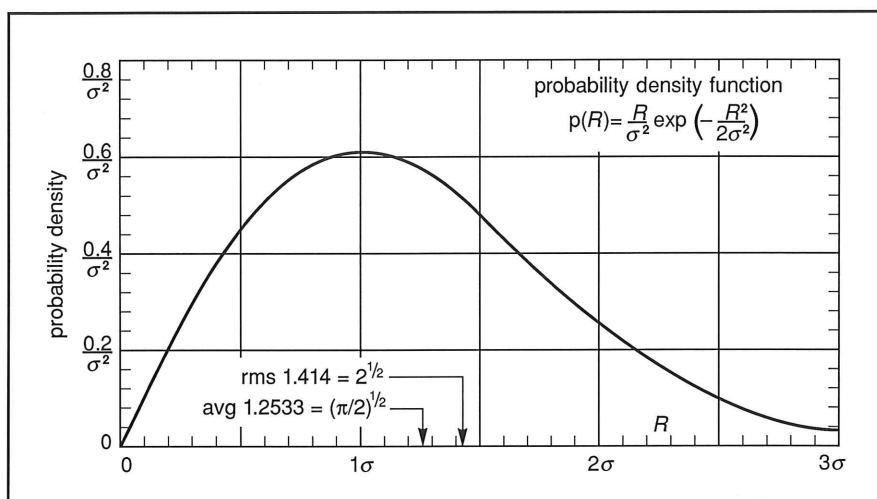


Figure 37. Theoretical Rayleigh distribution shape, approximated in 36.

Various information can thus be obtained, in addition to identification of random noise signals. Repetitive patterns will identify information bearing components intermingled with the random noise, for instance. However, one has to take care not to identify artifacts due to spectrum digitizer imperfections, as signal characteristics. The low level of intercepts at the 30% level in both figures 34 and 36, is suspicious. It could be a coincidence, but is more likely due to the digitizer dropping bits at that level. Further investigation to determine whether the suspicious feature is memory location dependent, or moves with the signal would be appropriate.

Noise Figure/Sensitivity. Thermal agitation of electrons in a resistance produces the random noise signal

known as thermal noise. The statistical RMS voltage squared equals $4RkTB$, and the power delivered to a matched load is $p=kTB$. With Boltzmann's constant $k=1.38 \times 10^{-23}$ joules per degree Kelvin, and temperature standardized at $T=290(\text{deg})K$ (16.84C), then $p=-174\text{dBm/Hz}$. This means that at room temperatures an ideal spectrum analyzer using a 1Hz bandwidth will not detect smaller signals than -174dBm (unless signal processing, e.g. averaging, is used to recognize the signal in the random noise). Using a 1kHz bandwidth the number is -144dBm , based on $10\text{Log}1000=30$, and so on. Of course real amplifiers introduce additional noise, and real filters and mixers introduce signal losses. The difference between the real and ideal situation is called the *noise figure*(F), with an

ideal device having a 0dB noise figure. The noise figure can be looked at as a front-end noise source in addition to the ideal, -174dBm/Hz . Sensitivity noise level and noise figure are simply related, just compare -174dBm to the actual noise level normalized to 1Hz. (An accurate comparison needs to use correct noise bandwidth, and 2.5dB correction.) Spectrum analyzers, especially at higher frequencies, exhibit significant front-end signal loss, resulting in poor noise figure. A sensitivity of -130dBm in a 10Hz bandwidth is good performance; but that is a 34dB noise figure. This is why some spectrum analyzers, such as the Tektronix 2710, include an internal low noise preamplifier. External preamplifiers are also frequently used. A preamplifier, no matter how perfect, will always reduce measurement dynamic range, because large signal drive level is reduced more than small signal sensitivity is improved. The preamplifier is, therefore, usually used for low level signal measurements and bypassed for large signals.

In spite of its generally poor noise figure, spectrum analyzers can be used to measure the noise figure of other, less noisy, items. Recall that noise figure equals the difference between actual noise and -174dBm/Hz . The equivalent input noise of an amplifier is the output noise less the dB gain. Thus, even a very low noise amplifier will produce enough output noise to be measured with a spectrum analyzer, if the gain is large enough. Additional amplifiers can be added in cascade if needed, and the first amplifier noise figure computed from $F(\text{total})=F(1)+[F(2)-1]/G(1)+[F(3)-1]/G(1)G(2)$, etc. Caution: use power ratios, not dBs. For example. We connect two identical, 25dB gain, amplifiers in cascade and measure a noise level of -120dBm/Hz . The gain of the cascade is 50dB, the input noise level is -170dBm/Hz , and the total noise figure is 4dB. The gain of the first amplifier is 25dB, or a power ratio of 316. Therefore, the contribution from the second amplifier can be neglected, and $F(1)=4\text{dB}$. With 4dB representing a 2.512 ratio, a complete computation gives: $2.512=F(1)+[F(2)-1]/316$. Since $F(1)=F(2)=F$, we find $F=2.507$, or 3.992dB.

Another procedure that eliminates calculating dBm/Hz noise levels uses an excess noise source, such as a noise diode. This is known as the Y factor method, because the ratio of output power with and without the excess noise source connected is designated "Y". It can be shown that, for a dB excess noise ratio E, $F=E-10\log(Y-1)$. For example: the displayed noise level changes by 11.7dB when a 15.4dB excess noise source is connected to the amplifier under test. 11.7dB represents a power ratio $Y=14.79$. Hence, $F=15.4-10\log(14.79-1)=4\text{dB}$. The advantage of this method is that corrections for noise bandwidth and detector functions, or general spectrum analyzer absolute level measurement errors, are eliminated.

Random Noise Volts. Random noise is usually measured in units of dBm/Hz. There are occasions however, when voltage rather than power is of interest. The normalization then is per root hertz, since power goes as voltage squared. One nice thing about normalizing to 1Hz is that the result is correct for both power and voltage, since one squared is still one. The simplest way to gain an understanding of how to deal with random noise in voltage units is through an illustrative example involving an actual measurement.

The noise properties of a new type of resistor was investigated using a spectrum analyzer. The spectrum analyzer was set for high input impedance (could be a high Z probe, the actual measurement used a Tektronix 7L5 set for 1 megaohm). The noise level shows -130dBv with input shorted, and -121dBv for a 100k(ohm) resistor. The theoretical open voltage level is

$$\sqrt{4RkTB}, \text{ or}$$

$$\sqrt{4 \times 10^5 \times 1.38 \times 10^{-23} \times 290 \times 10^3}$$

for a 1kHz bandwidth. This yields $1.26\mu\text{v}$, or -118dBv (dB with respect to one volt). Why do we measure -121, or 3dB better than a theoretical resistor at -118dBv?

The noise bandwidth is 750Hz, rather than the 1000Hz resolution bandwidth (1.25dB error). The spectrum analyzer shows a level 2.5dB below actual level. The correction for a $130-121=9\text{dB}$ difference between internal noise and measured noise is 0.6dB. The total difference is $1.25+2.5-0.6=3.15\text{dB}$. The 3dB discrepancy has been accounted for within the limits of measurement accuracy.

Noise Sidebands. No electronic circuit is totally free of noise. Random noise fluctuations in oscillators cause amplitude and phase modulation, resulting in spectrum sidebands around the carrier. These sidebands are usually known as *phase noise sidebands* due to the impact of frequency modulation on oscillator stability. The measurement is the same as for any carrier to noise determination, except that the noise spectrum is close in frequency to the carrier. Spectrum analyzers vary greatly in noise sideband performance. The spread in performance tends to increase with frequency because, among other factors, different spectrum analyzers operate on different local oscillator harmonic multiples. Performance degrades at the rate of $20\log N$, where N is the mixing harmonic multiple. For example, the Tektronix 2754P is specified at a noise sideband level of -103dBc/Hz, 30KHz offset from a 7GHz carrier. This number degrades to $-103+20\log 3=-93\text{dBc/Hz}$ at 21GHz, where $N=3$. By contrast, the 2782 provides -105dBm/Hz for only 10KHz offset at both 7 and 21GHz, because it uses fundamental mixing up to 28GHz.

Figure 38 shows the spectrum of a crystal oscillator derived 7GHz signal. The noise sideband at 30kHz offset measures -100.6dBc/Hz. This level is more than 10dB (one division at 10dB/div) above the lower trace, internal kTB noise. Therefore, no correction is necessary. However, spectrum analyzer sideband noise is specified at -103dBm/Hz, or only 2.4dB below that measured. The correction table then indicates a

3.5dB correction. This would be the worst case, because it is a rare spectrum analyzer where performance is not somewhat better than specified. Unfortunately, only a test of a very stable oscillator at the same frequency (7GHz) will indicate true spectrum analyzer performance. This is because the noise sideband is not there by itself, but due to modulation of a carrier. We can use the precision calibrator signal built into the spectrum analyzer in the absence of a clean signal at the desired frequency. Figure 39 shows -107dBm/Hz at 30KHz offset for the 100MHz calibrator. The spectrum analyzer will not be better than that at 7GHz. The correction factor, for a $107-100.6=6.4\text{dB}$ difference, is 1.1dB. The true level is somewhere in the range $-100.6-3.5=-104.1\text{dBm/Hz}$, and $-100.6-1.1=-101.7\text{dBm/Hz}$. An input level further above specification would not require any correction for spectrum analyzer generated phase noise.

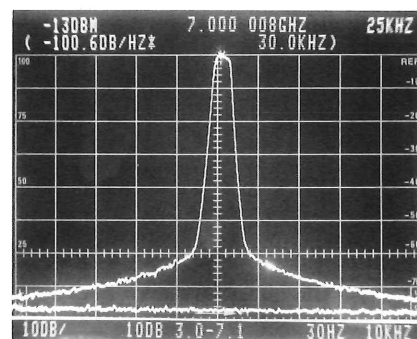


Figure 38. Phase noise sideband measurement.

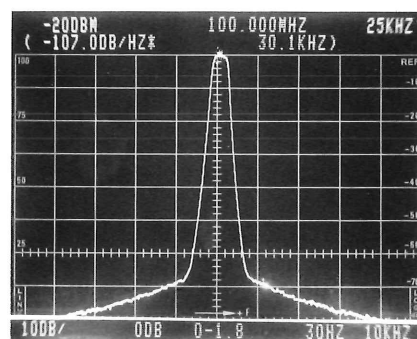


Figure 39. Phase noise sideband measurement.

Close-in sideband noise measurement calls for a spectrum analyzer with an especially narrow resolution bandwidth, and good enough local oscillator stability to permit such a measurement. The 2782 is such a spectrum analyzer. The instrument is specified at -85dBc/Hz at a 100Hz offset. Figure 40 shows a sideband level of 83dBc/Hz at 51.3Hz offset, using a 3Hz bandwidth filter. Note that the sideband noise level is only about 10dB worse at a 20Hz offset. Note also that for small offsets, the setting goes in 0.1Hz steps. Finally, note that the full screen sweep is 50 seconds, even though the video filter is set equal to the resolution of 3Hz . A video filter setting of 0.3Hz , barely suitable for noise averaging, would require a sweep time of 200 seconds. That is a long time to make a measurement. Instead the noise was averaged digitally at the marker location. This operation is actuated by the dBc/Hz function. It saves measurement time, and also permits averaging the noise without disturbing the signal. Most spectrum analyzers do not include this sort of, specialized, digital averaging.

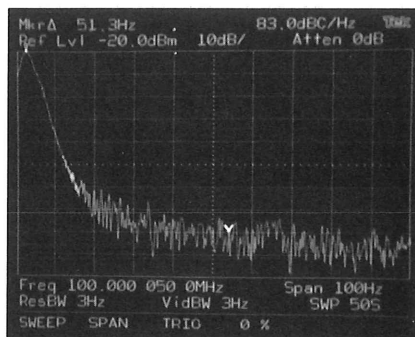


Figure 40. Close-in sideband noise measurement.

See Tektronix application note 26W-7047 for further details on noise sideband measurements.

Random Noise Measurement Reminders.

Remember the following when making random noise measurements with the spectrum analyzer.

- For best accuracy use the random noise bandwidth, rather than the resolution bandwidth.
- Remember to smooth the spectrum through averaging with the video filter, digitally or both.
- Remember the correction factors,

$$\sqrt{\frac{4}{\pi}} \text{ for linear mode and } 2.5\text{dB}$$

for logarithmic mode.

- Remember to keep the resolution bandwidth below $1/3$ the occupied bandwidth for accurate spectrum shape determination.

- Remember that noise powers are additive. Correct for spectrum analyzer internal noise.
- Remember that displayed noise level goes as 10LogB . Spectrum analyzer input circuitry is subjected to full input power level, and not just the displayed power level. Do not overdrive the input mixer.
- Remember that incoming noise and internal noise are both subject to the 10LogB behavior. Use of a narrower filter will not improve input noise measurement sensitivity.
- Remember that most new spectrum analyzers include normalization and error correction routines. Save time and use these. Do not create errors by correcting manually for what is corrected automatically.

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