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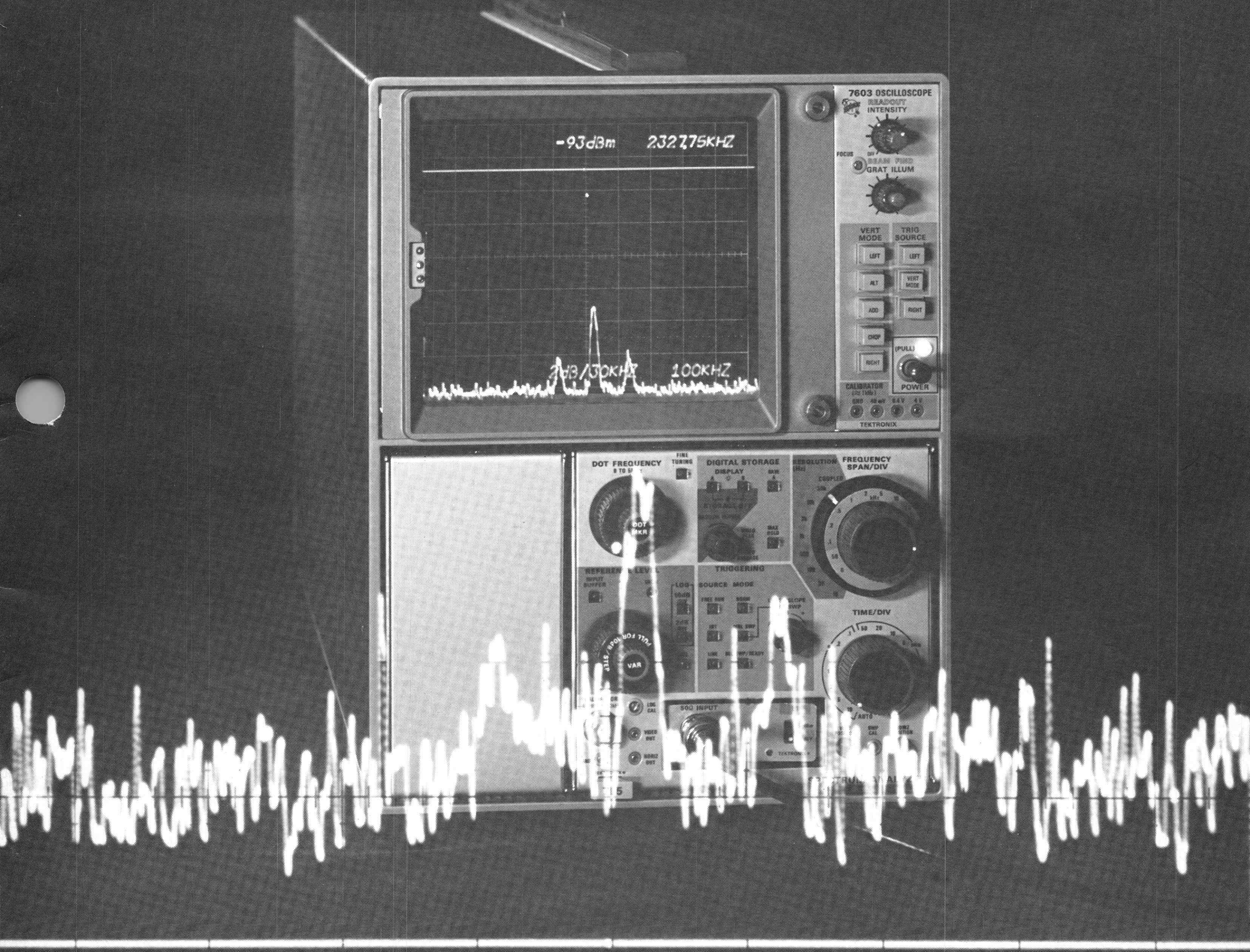
PART ONE: RANDOM NOISE



NOISE MEASUREMENTS USING THE SPECTRUM ANALYZER

Part One: RANDOM NOISE

By
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The need for noise measurement is as universal as the existence of noise itself. Whether the testing involves a signal-to-noise measurement, white noise loading in communications systems, oscillator spectral purity, or any other noise measurements, the basic need is still the same: *to measure noise power or noise power distribution as a function of frequency.*

The spectrum analyzer is an ideal instrument for such measurements. Described here are the details on how to correctly apply the spectrum analyzer to noise measurements. Where specific theory is applied by illustrations and examples, the TEKTRONIX 7L5 Spectrum Analyzer is used. Although some of the material in the text is specific to the 7L5 (such as digital averaging, or equivalent smoothing bandwidth), the basic theory is applicable to any spectrum analyzer regardless of make or model number.

This paper treats the subject in five sections. The first deals with effects due to the generally continuous spectral distribution of noise. This includes the need and means of determining effective noise bandwidths and the problems associated with overloading the input mixer. The second section deals with measurement errors and associated correction factors due to spectrum analyzer detector characteristics, logarithmic amplifier effects, and post-detection smoothing requirements. The third section describes the digital averaging system used in the 7L5, and gives the equivalent post-detection smoothing bandwidths for the system. The fourth section addresses the question of determining noise power distribution shapes, such as a noise notch used in noise loading tests. The paper concludes with a summary for quick review and reference.

I. Power per unit bandwidth

Random noise is described by a spectral power distribution. It is specified in power per unit bandwidth (dBm/kHz, mW/Hz, etc.). This has several effects on the use of the spectrum analyzer.

A. Noise Bandwidth

The random noise bandwidth of any filter is the average width of the power response curve (voltage squared). This average width is equal to the bandwidth of a theoretically ideal square-shaped filter that has the same area and amplitude as the actual power response curve. The random noise bandwidth is easy to measure, as shown here.

Figure 1 is a linear response curve (voltage) of the 3-kHz resolution bandwidth position for a 7L5 spectrum analyzer. We now plot voltage squared (vertical) versus frequency (horizontal) (in figure 2). The area under the curve is 29 squares and the height is 6.4. This yields an average frequency width:

$$\frac{29\,(\text{div})^2}{6.4\text{ div}} \times 0.5\,\frac{\text{kHz}}{\text{div}} = 2.27\text{ kHz random noise bandwidth}$$

For comparison, observe that the specified resolution bandwidth in Figure 1 is a 3-kHz 6-dB bandwidth and 2.2-kHz 3-dB bandwidth. Thus 1.03 and 0.76 are the ratios of noise bandwidth to 3-dB and 6-dB bandwidths respectively. Other bandwidths on the 7L5 were measured as well. The ratios are given in table 1. A good rule of thumb for conversion to noise bandwidth is a ratio of 1 for the 3-dB bandwidth and about 0.75 for the 6-dB bandwidth.

Specified Resolution Bandwidth	30 kHz	10 kHz	3 kHz	1 kHz	300 Hz	100 Hz
B_n/B_3	0.95	1.02	1.03	1.1	1.05	1.0
B_n/B_6	0.78	0.85	0.76	0.73	0.74	0.70

Table 1. Ratios of noise bandwidth to 3-dB and 6-dB bandwidths for a 7L5.

These numbers are somewhat smaller than those theoretically computed for an array of synchronously tuned filters. It has been shown, for example, that the noise bandwidth is 1.155 wider than the 3-dB bandwidth for a cascade of three synchronously tuned filters.[1] However, these filters consist of unconventionally coupled multiple stages that do not yield a true Gaussian response.

[1] *T. L. Martin, Jr., Electronic Circuits pp. 330 Prentice Hall, 1955.*

So we suggest the following procedures with respect to bandwidth when you use the 7L5 to measure noise power.

1. For a quick-look approximation, use a noise bandwidth that is 0.75 of the specified resolution bandwidth. Note that Tektronix specifies the resolution bandwidth at 6 dB down rather than at 3 dB down. This and other aspects of spectrum analyzer resolution are discussed in the literature.[2]
2. An alternate quick-look method when you are measuring in dBm is to use the specified resolution bandwidth and add a correction factor of 1.25 dB. This corrects for the 0.75 factor discussed in (1).
3. Typical spectrum analyzer resolution bandwidth specifications are ± 20%. For better accuracy without much extra work, measure the actual 3-dB or 6-dB bandwidth and multiply by the appropriate factor, 1 or 0.75 respectively for the 7L5 spectrum analyzer.
4. For greatest accuracy, determine the random noise bandwidth as illustrated in figures 1 and 2. Note that this need not be done every time the spectrum analyzer is used. Once the random noise bandwidth for a given resolution bandwidth setting has been established for a particular analyzer it can be used over and over again.

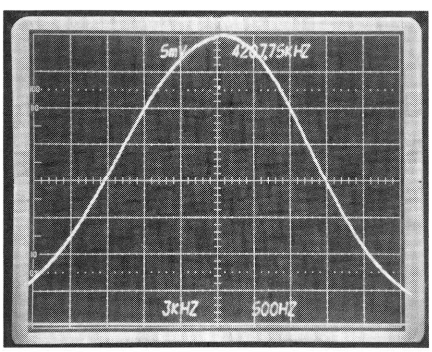


Figure 1. Linear (voltage) resolution bandwidth response.

Now that the random noise bandwidth is known, the noise power of interest can be determined. Simply divide the observed noise power by the random noise bandwidth used to determine the signal noise power (in watts/Hz, dBm/MHz, etc.). Consider, for example, a random noise power level of −40 dBm using the 1-kHz resolution bandwidth of a 7L5. Using the alternate quick-look method in (2), add a correction factor of 1.25 dB so that the true noise power is −38.75 dBm/kHz. We could also measure the effective random noise bandwidth, as described in (4). Suppose that the result is:

$$B_n = 730 \text{ Hz}.$$

Hence the true noise power level is:

$$\frac{10^{-4} \text{ mW}}{0.730} = 137\ \mu\text{w/kHz}\ (-38.63 \text{ dBm}).$$

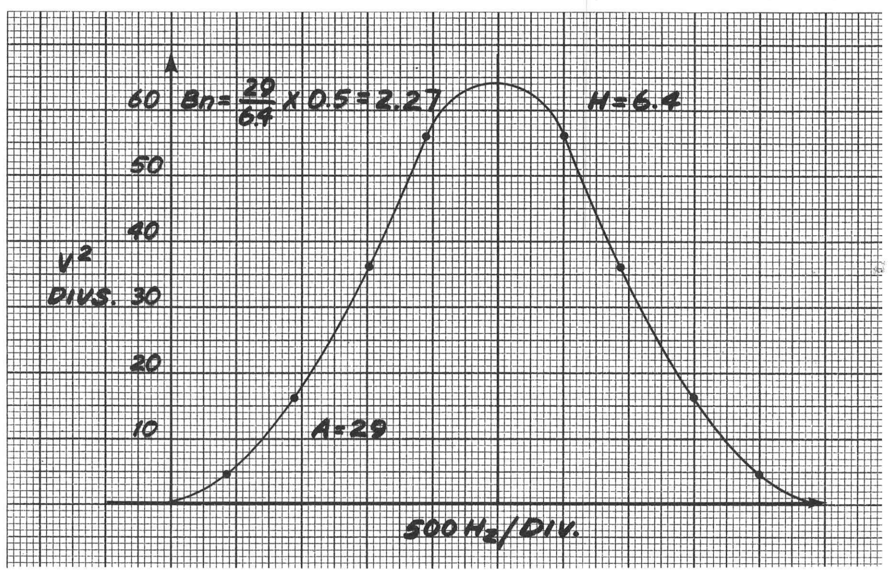


Figure 2. Power (voltage squared) resolution bandwidth response.

B. Input Power

While the indicated noise power is dependent on the resolution setting of the spectrum analyzer, changes in resolution setting have no effect on the noise power impinging on the input mixer or attenuators. This input noise power level depends, rather, on the overall noise frequency distribution. For wideband noise sources, the distribution can extend the full width of the spectrum analyzer's input filter bandwidth, the widest frequency range of noise power that can get to subsequent circuitry. For the TEKTRONIX 7L5, the input filter bandwidth is 5 MHz.

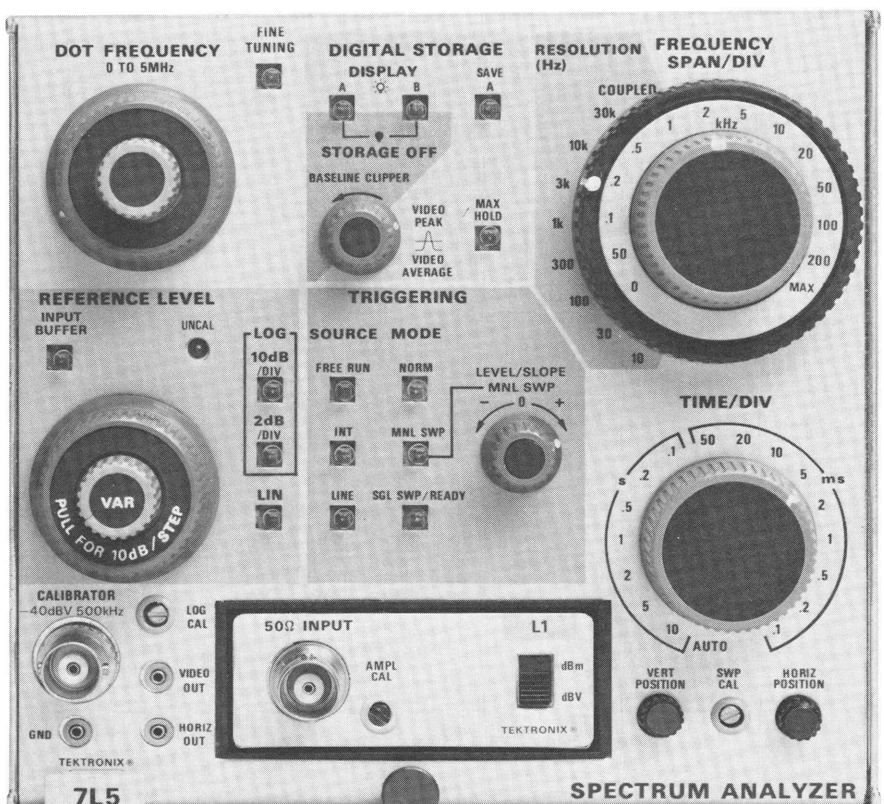
[2]*Engelson, "Understand Resolution for Better Spectrum Analysis," Microwaves, December 1974.*

Using the widest resolution bandwidth of 30 kHz (with a noise bandwidth of about 22.5 kHz), we have a possible input power level of $5 \times 10^6 / 22.5 \times 10^3$, or 222 times that indicated on the crt. For the 7L5, then, input power can be as much as 23.5 dB (222 times) greater than that measured at 30-kHz resolution, and the difference increases at 10 dB per 10 times reduction in resolution bandwidth and at 5 dB per 3 times reduction in resolution bandwidth. Hence, the input power could be as much as 58.5 dB more than that observed at 10-Hz resolution bandwidth.

Large front end input power can cause several problems:

1. Damage: Power input levels greater than +10 dBm can cause permanent damage to the input mixer, and levels above +20 dBm can cause permanent damage to the input attenuators. A small level on screen is no assurance that the front end is safe.
2. Linear Operation: Input mixers start going non-linear at an input power level of about −10 dBm. This level should never be exceeded for accurate absolute noise level measurements. At 30-kHz resolution bandwidth setting, the mixer remains linear at a crt indication of up to −33.5 dBm. At a 10-Hz resolution bandwidth setting, the linear limit is reached at −68.5 dBm.

   A simple way to ensure that the front end is not being overloaded is to change the input level by switching input attenuation in or out. The front end is not overloaded if the measured power follows the attenuation change. The input buffer control in the 7L5 is especially suited for this application. This control adds 8 dB of front-end attenuation while simultaneously adding 8 dB of if gain. The reference level and indicated signal level should therefore remain unchanged, provided that the front end is operating in its linear mode.*
3. Intermodulation (IM): Sometimes it is necessary to observe low noise levels in one frequency range and high levels in another range. Intermodulation noise products from the high level portion must not be allowed to obscure the low level input. The 7L5 intermodulation products are at least 75 dB down for −30-dBm input to the mixer. If the input noise were to extend over the full 5-MHz frequency range, the crt display would show 23.5 dB less, or −53.5 dBm at a 30-kHz resolution bandwidth setting. With a sensitivity of −105 dBm, we have a dynamic range of 51.5 dB. The dynamic range can be improved by limiting the input noise bandwidth to less than the full 5 MHz. Activating the input buffer is a simple way of checking whether IM products are being produced by the spectrum analyzer.



The effect of the input buffer (and, in fact, most of the measurements described in this paper) is based on the assumption that the noise signal to be measured has a much higher level than the spectrum analyzer's internal noise. In those rare cases where this assumption doesn't hold, the effect of internal noise must be subtracted.

II. Detector and Smoothing Filter Characteristics

Envelope Detectors: Random noise is described statistically by a Gaussian distribution. However, by the time the noise signal gets to the detector, it has passed through several filters, so we are dealing with a band-limited Gaussian process with a Rayleigh distribution: random noise within an envelope that looks like a distorted random sinusoid. The spectrum analyzer's envelope detector displays the detected envelope, as shown in figure 3. Post-detection filtering or smoothing is then used to establish the average value of the Rayleigh envelope distribution which is displayed as a smooth line on the crt. It has been shown that the average value of a Rayleigh distribution is $\sigma\left(\frac{\pi}{2}\right)^{1/2}$, while the rms value of such a distribution is $\sigma(2)^{1/2}$, where σ is the variance.[3] Hence, the average, or smoothed (video filtered), value displayed by the spectrum analyzer is $\left(\frac{4}{\pi}\right)^{1/2}$ or 1.13 (1.05 dB) less than the rms value.

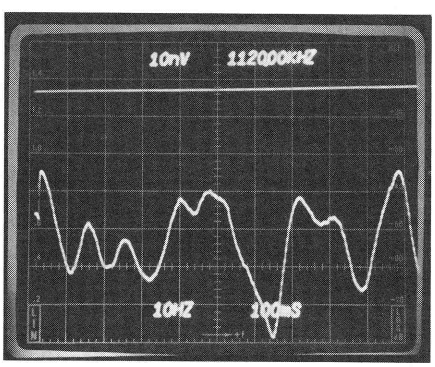


Figure 3. Detected noise envelope shape.

[3]*"Reference Data for Radio Engineers," ITT, 4th ed. pp. 991.*

To convert the measured average value to the desired rms value, multiply noise voltages measured in the linear mode by a factor of 1.13.

Log Circuits: Logarithmic processing before detection further distorts the noise distribution and introduces the possibility of additional measurement errors. It has been shown that the theoretical noise measurement error in the logarithmic mode is 2.5 dB, which takes into account the 1.05-dB error of the detector.[4] The user should add 2.5 dB to all noise level measurements performed in the logarithmic mode.

The additional error introduced by logarithmic processing is quite easy to verify. Figure 4 is a dual-trace display of a noise and cw signal of equal display level in the linear mode. Figure 5 shows the same signals, but this time the 7L5 spectrum analyzer is operating in the 2-dB/div logarithmic mode. Here the noise (the wiggly line) is about 1.4 dB below the level of the sine-wave display amplitude. The theoretical difference should be: 2.5 - 1.05 = 1.45 dB.

Smoothing Filter: The rate at which the Rayleigh distribution envelope varies depends on the bandwidth of the filter the wideband noise passes through. The narrower the bandwidth, the slower the variation of the envelope. In figure 3 seven random periods seem to occur in a 1-s total time interval. This 7-Hz frequency is about the same as the 3-dB bandwidth of the 10-Hz resolution filter used.

Thus, since the average frequency of the function to be smoothed is of the same order as the resolution bandwidth, the smoothing (or video) filter must get progressively narrower as the resolution bandwidth is made narrower. It has been shown

[4]*"Spectrum Analysis — Noise Measurements," Hewlett Packard, Application Note 150-4, January, 1973.*

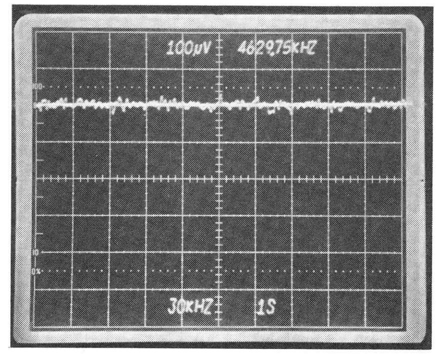


Figure 4. Noise and equal display cw signal in linear display mode.

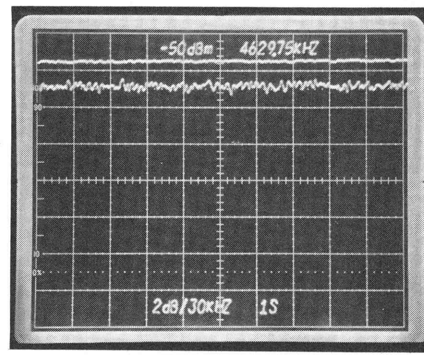


Figure 5. Noise and equal display cw signal in logarithmic display mode.

that the maximum measurement error due to insufficient post-detection smoothing is $\left(\frac{2B_v}{B_n}\right)^{1/2}$, where B_v is video filter bandwidth and B_n is noise bandwidth.[5] A post-detection smoothing filter bandwidth one-hundredth as wide as the noise bandwidth could cause a measurement error of up to 14%. Post-detection bandwidth should, therefore, be kept as narrow as possible. A bandwidth ratio of one hundred to one is a reasonable goal.

[5]*Sutcliffe, "Relative Merits of Quadratic and Linear Detectors in the Direct Measurement of Noise Spectra," The Radio and Electronic Engineer, February, 1972.*

III. Digital Averaging

Unlike most spectrum analyzers, the 7L5 does not have a post-detection "video filter." Instead, it has a digital averaging function. What does this mean? After incoming signals have been processed by resolution bandwidths, logarithmic amplifiers, etc., the signals are peak detected by an envelope detector and sampled for storage in an electronic memory. Samples are taken once every 9 μs and loaded into 512 memory locations equally spaced across the crt. The number of samples per memory location is a function of sweep time. Thus, at 100 ms/div (1 s full-screen), there is a total of 1.1×10^5 samples per 512 locations, or at least 200 samples per memory location. The average value is then computed and a single average amplitude displayed for each of the 512 locations.

What smoothing bandwidth does this kind of averaging represent? Consider the simplest example, a sinusoid with zero crossings that coincide with each of the 512 memory locations. This gives a half sine-wave that occupies one memory location and averages out to $\frac{2}{\pi}$, or a 3.9-dB reduction in amplitude.

At a 50 ms/div sweep time, for example, the time interval between memory locations is

$$\frac{50\ \text{ms/div} \times 10\ \text{div}}{{}^*500\ \text{locations}} = 1\ \text{ms/location}.$$

It is easier to use 500 rather than 512 locations. Actually, the whole calculation is approximate since the 512 locations cover the full horizontal width, including about 0.5-division of overscan. The number of locations across the 10-division graticule is therefore less than 512.

At 1 ms per half cycle we have a 3.9-dB bandwidth of 500 Hz. A more complicated calculation yields a 3-dB bandwidth of 425 Hz. Table 2 lists other post-detection bandwidths.

Time/ Div	10 s	5 s	2 s	1 s	0.5 s	0.2 s	0.1 s	50 ms	20 ms	10 ms	5 ms
Digital Averaging Filter Bandwidth	2.5 Hz	5 Hz	12.5 Hz	25 Hz	50 Hz	125 Hz	250 Hz	500 Hz	1.25 kHz	2.5 kHz	5 kHz

Table 2. Equivalent Post-Detection Bandwidth with Digital Averaging

To get a better feel for what these numbers mean, consider the following illustrations:

Figure 6 shows the spectral display of a carrier with 100% amplitude modulation at a 500-Hz rate. Figure 7 shows the carrier with the modulation removed and the resolution bandwidth increased to 10 kHz. Note that the carrier level has not changed. Figure 8 is the same as figure 7 except that the modulation has been turned on. With the 500-Hz sidebands unresolved and the spec-

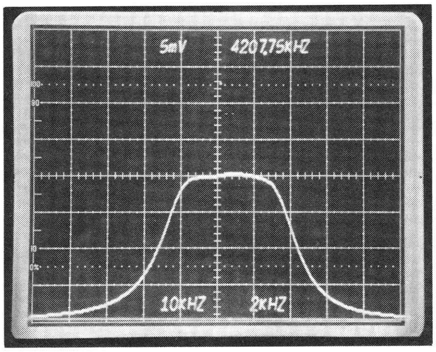


Figure 7 Same carrier as figure 6; no modulation and wider resolution bandwidth.

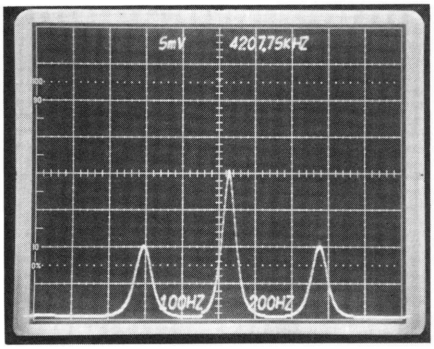


Figure 6 100% am with sidebands resolved.

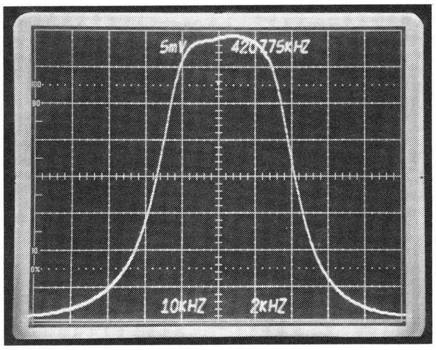


Figure 8 Same as figure 6; sidebands are not resolved, peak display mode.

trum analyzer set to display the output of the peak detector, the output is twice that of the carrier alone, as might be expected for 100% am. Figure 9 is an "averaged" figure 8. The 500-Hz modulation has been averaged out and we are back to the carrier that corresponds to the average value of an am waveform. Figure 10 shows the peak-detected 500-Hz modulation in more detail, while figure 11 shows the same modulation average with an equivalent 500-Hz filter. Note that the displayed level is down from about 8 div to 5 div, or 4 dB, which is very close to the theoretical 3.9 dB computed previously.

Two final points about equivalent post-detection bandwidths: First, the 7L5 can split the memory into two sections that consist of 256 sample locations each. This doesn't change the equivalent post-detection bandwidth from the values computed for 512 points with both memories interlaced, because the detected signal is always sampled at 512 positions, with half the positions not displayed when either half of the memory is turned off.

And second, it is possible to get some strange-looking displays when averaging coherent functions such as sine waves. This happens when the time per division is such that an integral multiple of signal cycles fills one memory slot (1/512th of the screen). The average value of a full sine wave is zero, so the equivalent post-detection bandwidth appears to be near zero. Jitter (in the rate of sampling and in the incoming waveform) will cause the equivalent post-detection bandwidth to appear to jump between zero and some finite value, causing the displayed waveform to jump around in amplitude level. The simple remedy is to go to a different time/div.

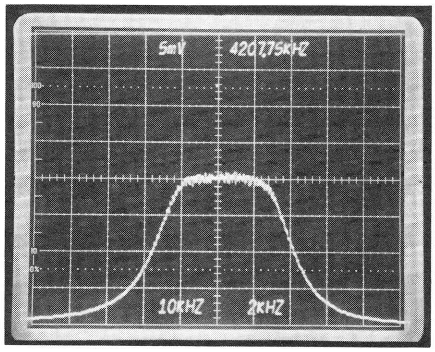


Figure 9 Same as figure 8; sidebands are not resolved, average display mode.

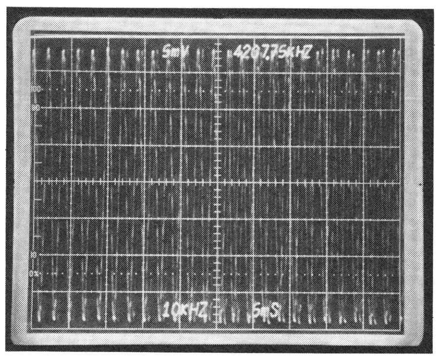


Figure 10 Zero span (time domain) display of detected modulation, no post detection smoothing.

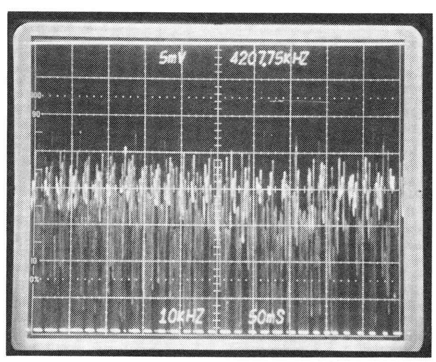


Figure 11 Zero span (time domain) display of detected modulation, with post detection smoothing.

IV. Choice of Resolution Bandwidth

Internal noise generated by the spectrum analyzer responds to changes in resolution bandwidth settings the same way as an externally applied noise signal. This means that the amplitude difference between a displayed external noise signal and the spectrum analyzer noise floor will remain constant regardless of resolution setting. In other words, sensitivity is not affected by changes in resolution. Ideally, then, the spectrum analyzer should be operated at the widest possible resolution bandwidth for the greatest smoothing effect from post-detector filtering. But this is not possible when the measurement involves determining a noise power distribution shape: in noise loading tests in communication systems; for example, where a notch shape within a noise distribution or a filter shape filled with noise must be displayed.

Ideally, the resolution bandwidth should be so narrow that the shape to be measured isn't distorted, but this is not always possible. Figure 12 shows the effect of resolution bandwidth setting on such a measurement. This is a four-trace display of a 20-kHz wide filter driven by white noise, with the result displayed on a 7L5 spectrum analyzer. The 7L5 resolution bandwidth was set at 30 kHz, 10 kHz, 3 kHz, and 1 kHz respectively. Spectrum analyzer gain was adjusted to produce equal amplitude responses at each of these resolution bandwidth settings. From the photo, it is clear that there is virtually no change between the 1-kHz and 3-kHz setting, and that there is a very small error at the 10-kHz position. The 30-kHz resolution bandwidth is obviously unusable for observing a 20-kHz wide noise burst. This photo illustrates a basic rule of thumb: the resolution bandwidth should be about one-third to one-tenth as wide as the noise shape to be observed. The user needs to establish a compromise within these limits to get the best shape definition, but should not choose such a narrow bandwidth that post-detection smoothing becomes a problem.

As an example, consider a noise loading test that calls for the definition of an 800-Hz wide notch. A 300-Hz resolution bandwidth with 2.5-Hz post-detection smoothing (10 s/div averaging) is probably the best compromise. The only other possibility is a 100-Hz resolution bandwidth setting for slightly better notch shape definition, but this will result in degraded trace smoothing.

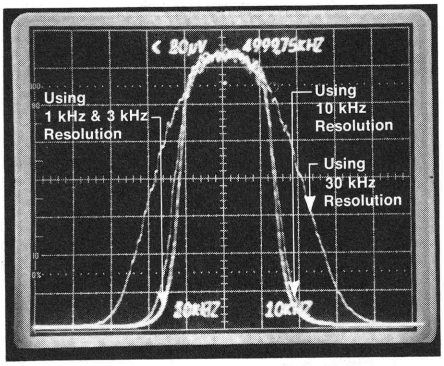


Figure 12. 20-kHz noise shape as resolved using 1-kHz, 3-kHz, 10-kHz, 30-kHz resolution bandwidth position.

V. Summary

When you make random noise measurements with the 7L5 spectrum analyzer, keep the following points in mind:

1. Multiply the resolution bandwidth (6 dB down) by 0.75 to get an approximate random noise bandwidth. Measure the actual random noise bandwidth for accuracy.
2. Don't drive the front end beyond its linear range. The front end can see as much as 5 MHz/B_{noise} greater noise power than you observe on the display. Use the input buffer control as an easy check for linear operation.
3. Random noise power is greater than the displayed measurement by a factor of 1.13 in Lin and 2.5 dB in Log because of envelope detector characteristics.
4. Post-detection smoothing filter bandwidths should be less than one-hundredth of the resolution bandwidth for accurate measurements. Possible percentage error due to insufficient post-detection smoothing (video filtering) is given by
$$\left(\frac{2 B_v}{B_n}\right)^{1/2}$$
5. The 7L5 uses digital averaging so that post-detection smoothing bandwidth is a function of time interval per sampling memory location. This time interval is determined by the time/div setting as shown in table 2.
6. From (4) and (5), it follows that accurate random noise measurements can be performed down to resolution bandwidth settings of 300 Hz. Marginal measurements can be performed at a resolution bandwidth of 100 Hz. Measurements at resolution bandwidths below 100 Hz will be considerably in error.
7. Notch or filter shape definition needs a resolution bandwidth less than one-third the shape being observed. The measurement will not be improved by using a ratio of more than ten to one. According to the principle in (6), notch and filter shapes greater than about 1-kHz bandwidth can be accurately defined; shapes down to about 300-Hz bandwidth can be marginally defined; shapes having less than 300-Hz bandwidth cannot be measured usefully with the 7L5.
8. All spectrum analyzers, not just the 7L5, are subject to these measurement limitations.

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