

Pulse Reflection Measurement  
of Transmission Line Impedance  
and Discontinuities

Gordon D. Long

Tektronix, Inc.

P.O. Box 500

Beaverton, Oregon

Copyright (c) 1962 by Tektronix, Inc., Beaverton, Oregon. Printed in the United States of America. All rights reserved. Contents of this publication may not be reproduced in any form without permission of the copyright owner.

## Pulse Reflection Measurement of Transmission Line Impedance and Discontinuities

Pulse reflection measurements can be performed with a high degree of resolution using sampling type oscilloscope systems. Utilizing the dynamic range of several volts and sensitivity of millivolts, one can resolve voltage reflections to better than 1 part in  $10^3$ . Fractional nanosecond time resolution enables separation of points (on a transmission line) only several centimeters apart.

Transmission line reflections are caused by impedance discontinuities along the line. The following sections give typical methods used to correlate an observed reflection with a line discontinuity, as well as describe completely a system capable of 0.1% voltage reflection measurements.

### Description of System: (See equipment list and Figures 1 and 2)

In the systems diagrammed below, the Type 110 or fast pulser generates a fast rising step which has a flat top for the duration of the pulse. The series combination of 100 pf and 50-ohm at the end of the charge line in Figure 1 provides an exponential return to the initial voltage. Thus, the driving pulse has only one fast rise transition to interrogate discontinuities. The single fast transition will reduce the confusion factor associated with interpretation of waveforms as well as reducing by two all extraneous system reflections. Triggering energy is extracted in the trigger takeoff of the Type 110 or from a Trigger Takeoff Transformer and is delayed by 60 to 70 nSec before reaching the external input of the Type 5T1. This delay is needed to place the signal reflected from the test point in the time window of the Type 5T1/4S1.

(An optional triggering scheme is to locate a Trigger Takeoff Transformer near the test point. This is not suggested as the transmission loss of the trigger device is included twice.)

The drive pulse then passes through a X2 Attenuator and into a 50-ohm "T" section. One-half of the transmitted energy then travels toward the test point through the Type 113, which delays the desired reflection well past the exponential decay of the drive pulse, and the 50-ohm impedance standardizing air lines. The reflection from the test point returns to the "T" connector to mix with the drive pulse. The combined signal (Figure 3a) proceeds to the Vertical Sampling Unit (4S1 or X4S2) input through an Attenuator which further reduces small reflections associated with the Vertical Sampling Units. Note that system reflections of 1%, normally considered small for many uses, can be quite disastrous when 0.1% resolution is desired.

#### System Calibration:

To standardize the amplitude of the reflection from the unterminated (or shorted) test point: Set the Type 4S1 or X4S2 step attenuator to 200 mv/cm; then set the pulse amplitude with pulser (Type 110 or 109) and/or with 50-ohm line attenuators (if using fixed amplitude pulser) for 10 centimeters (or some integral number larger) vertical deflection. Check to see that the open and shorted test point reflections are equal in amplitude but of opposite polarity (See Figure 3a). The reflected pulse will have a waveform as indicated in Figure 3b. This pulse consists of a fast step ( $\sim 400$  pSec or  $\sim 200$  pSec with X4S2) at the leading edge with amplitude A. The distributed losses of the coax used in the system cause a "dribble-up" between amplitudes A and B. This effect is less than 10% of the pulse amplitude and lasts for approximately 5 nSec. For accurate impedance measurements (terminations, sections of coax cable, etc.), amplitude B should be used as the standard pulse height. When small discontinuities are being observed, amplitude A is used as the standard.

In either case, establishment of a known amplitude when at 200 mv/cm will enable vertical magnification of the display at least 1000 times. (For 10 cm initially, rotate attenuator to 2 mv/cm.) When so expanded, each cm of displacement represents 0.1% voltage reflection. Smoothing can be usefully employed when operating at full resolution.

The final steps of calibration are checking the sweep rate and measuring the risetime of the fast transition of the reflected pulse (10 to 90% of amplitude A, as in Figure 3b). This is the system risetime  $T_r$  and is employed in subsequent calculations.

#### Impedance Measurements:

An impedance of  $Z_L$  at the test point will produce a voltage reflection given by:

$$\rho = \frac{Z_L - 50}{Z_L + 50} \quad \text{for standard 50-ohm system.}$$

If the voltage reflection coefficient  $\rho$  is small  $Z_L$  will be close to 50-ohms and the difference  $\Delta$  in ohms can be expressed as:

$$\begin{aligned} \Delta &= 100\rho \\ \text{we have set } Z_L &= 50 + \Delta \text{ in the above so that } \rho = \frac{\Delta}{100 + \Delta} \\ \text{if } \Delta \ll 100, \text{ then } \rho &\approx \frac{\Delta}{100} \end{aligned}$$

This is accurate to better than 10% for  $\rho < 0.09$  (9% reflection).

For larger values of  $\rho$  :

$$\Delta = \frac{100\rho}{1 - \rho} \text{ ohms} \quad (\text{for 50-ohm system})$$

The above method is valid for resistive terminations and for sections of transmission line which are at least 0.5 nSec in electrical length. For measurements of this type, amplitude B of the drive pulse should be standardized.

The above discussion is valid for nearly any combinations of  $Z_L$  and  $Z_0$  (50-ohm is most useful for  $Z_0$ ). The best resolution is possible when  $Z_L$  and  $Z_0$  are nearly equal in that a substitution method may be employed. This is done by comparing two impedances  $Z_L$  at the same time-reflection length past the end of the  $Z_0$  section. For purposes of accuracy, one of the  $Z_L$ 's employed should be a termination resistance which has been measured to desired accuracy with a DC bridge. See Appendix A for discussion of transferring DC measurement to nanosecond pulse measurement.

#### Location of Discontinuities:

A length  $L$  along the reflection stub will have a delay time of  $\frac{L}{V_p}$  where  $V_p$  is the propagation velocity. Since the reflection must travel back through the reflection stub, the oscilloscope will display the time between discontinuities in the stub as  $\frac{2L}{V_p}$ ; where  $L$  is the physical separation. Thus, positions which are 30 cm apart in air dielectric coax line will appear 2 nSec apart in the reflection display. If it is desired to locate accurately the position of a small lumped discontinuity, one can usually find a way to add a small reflection to the system near the discontinuity. If the added reflection is of opposite polarity and of nearly equal magnitude to the unknown disturbance, the location of the unknown will be adjacent to the position of the added reflection which results in a minimum over-all reflection. This is a null method and can be done quite accurately. If the position is not correct, the resulting reflection will generally have zero average deviation, but will not be minimized (See Figure 4). To cancel completely, the magnitude and position must both be correct simultaneously.

### Measurement of Small Discontinuities:

In many instances of pulse reflection tests, the major concern is with discontinuities which are both short in length and small in magnitude. Since the reflection from this class of discontinuity is very short compared to the system risetime, the reflection is reduced in amplitude (as shown below) and "smeared out" in time (to approximately  $T_r$ ) by integration in the test system. Locating the position of discontinuities has been covered above.

In a section concerning impedance determination of a connector in a transmission line, Lewis & Wells (1) state that given a small but constant impedance deviation, and a double transit time very short compared to the test system risetime, the observed voltage reflection coefficient will be less than the actual  $\rho$ .

Stated simply:

$$\rho_{obs} = \frac{2T}{T_r} \rho_{act} \quad \text{Equation 1}$$

where:  $\rho_{obs}$  is the observed voltage reflection coefficient.  
 $2T$  is the double transit time of the discontinuity.  
 $T_r$  is the test system risetime.

$$\rho_{act} = \frac{Z_s - Z_o}{Z_s + Z_o} \begin{cases} \text{is the actual voltage reflection} \\ \text{coefficient of the section in error.} \end{cases}$$

$Z_s$  is characteristic impedance of section in error.

$Z_o$  is system characteristic impedance.

In this relationship, the effect of test system integration is obvious.

In another reference (2) Sugarman has derived a relationship between  $\rho$  (observed) and a single lumped reactive discontinuity in a continuous transmission line, where the observing system has a limited risetime.

## Equation 2a

$$\rho_{obs} = \frac{C Z_0}{2 T_r}$$

where: C is magnitude of lumped capacitance shunted to ground in a continuous system of impedance  $Z_0$ .

$T_r$  is system risetime.

Note that the time constant of the reflection is  $\frac{C Z_0}{2}$  and that again this time will be short compared to  $T_r$ . As with Lewis & Wells' equation, the amplitude of  $\rho_{obs}$  is directly dependent on the ratio of time duration of discontinuity to system risetime.

Use of one of the standard transmission line relationships  $\left(\frac{L}{C} = Z_0^2\right)$ , will convert Sugarman's derivation to:

## Equation 2b

$$\rho_{obs} = \frac{L}{2 T_r Z_0}$$

The approximation when the discontinuity is a lumped series inductance.

To return to the real world of physical discontinuities, an impedance error in a transmission line will usually have a finite length. A pure, lumped reactive discontinuity does not exist. In spite of this reality, it is convenient to refer to short lengths of line (short compared to the system risetime) as having an equivalent lumped reactance. If the impedance of a particular section is low, that portion will exhibit an equivalent shunt capacitance ( $\rho$  will be negative). If impedance is high, it will be an equivalent series inductance ( $\rho$  will be positive).

Equating the above quoted approximations will show the equivalent lumped reactance of a short discontinuity in a line of  $Z_0$ .

for  $\rho$  negative: (Capacitance)

$$C = - \frac{4T}{Z_0} \left( \frac{Z_s - Z_0}{Z_s + Z_0} \right) \quad \text{Equation 3a}$$

for  $\rho$  positive: (Inductance)

$$L = 4TZ_0 \left( \frac{Z_s - Z_0}{Z_s + Z_0} \right) \quad \text{Equation 3b}$$

Proof that the two quoted approximations for  $\rho_{obs}$  are equivalent appears in Appendix B.

To apply the above to actual measurements, first obtain a value for  $\rho_{obs}$  from the test system and discontinuity, then use Equation 2a or 2b to obtain the magnitude of the equivalent reactance.

#### Examples of High Resolution Measurements:

The photos illustrate typical displays appearing on the described systems. Captions give details about sensitivity, type of test and system employed. With experience, tests may be run in a short time because all the pertinent data are obtained with one system set-up and pulse display. Pulse reflection tests have been known for giving information rapidly; this article has illustrated that a carefully assembled system is fully capable of good resolution and accuracy. The development of sampling oscilloscopes has made this reflection system possible.



## Appendix A

Estimating transient behavior of an assembly containing a terminating resistor. This discussion assumes that the following static and dynamic conditions are fulfilled:

1. Resistance change with temperature or voltage changes is smaller than accuracy desired.
2. Resistance of assembly holding R is negligible.
3. R is physically small.

The problem is that the transmission line in the assembly might not have the same characteristic impedance as does the termination R. If such is the case, then a length of time will be required to establish the ohmic value R to the degree of required accuracy.

The time required:

$$t \cong -2T \left( \ln \left[ \frac{\frac{A}{R - Z_s}}{R + Z_s} \right] + 1 \right)$$

where:

A                      desired accuracy.

$\frac{R - Z_s}{R + Z_s}$               reflection coefficient of assembly.

$R = Z_o$               is termination standard.

2T                      is double transit time of assembly.

This is because the reflection from a discontinuity settles down in step type approximation to an exponential decay.

## Appendix B

Transmission line characteristic equations.

$$Z_o^2 = \frac{L_o}{C_o}$$

$$V_p^2 = \frac{1}{L_o C_o}$$

$L_o, C_o$  are distributed line parameters.

In terms of  $Z_o, V_p$  and length  $l'$ .

$$L_o = \frac{l' Z_o}{V_p}$$

$$C_o = \frac{l'}{Z_o V_p}$$

For lines with  $l'$  and  $V_p$  equal, but with dissimilar impedances  $Z_o$  and  $Z_s$  the distributed parameters are:

$$L_o = \frac{l' Z_o}{V_p}$$

$$L_s = \frac{l' Z_s}{V_p}$$

$$C_o = \frac{l'}{Z_o V_p}$$

$$C_s = \frac{l'}{Z_s V_p}$$

Taking differences in distributed parameters:

$$C_s - C_o = \frac{l'}{Z_s V_p} - \frac{l'}{Z_o V_p}$$

$$C_s - C_o = \frac{l'}{V_p} \left( \frac{1}{Z_s} - \frac{1}{Z_o} \right)$$

and:

$$L_s - L_o = \frac{l'}{V_p} \left( Z_s - Z_o \right)$$

Now use Sugarman's approximation (Equation 2a and 2b of article) to find  $\rho_{obs}$  for both differences.

For C:

$$\rho_c = \frac{(C_s - C_o) Z_o}{2 T_r}$$

$$\rho_c = \frac{1' Z_o}{2 V_p T_r} \left( \frac{1}{Z_s} - \frac{1}{Z_o} \right)$$

For L:

$$\rho_L = \frac{L_s - L_o}{2 T_r Z_o}$$

$$\rho_L = \frac{1'}{2 T_r V_p Z_o} (Z_s - Z_o)$$

We now have expressions for the observed reflection coefficients for both inductive and capacitive effects. These are the disturbances required on the line of  $Z_o$  to form a short section of different impedance  $Z_s$ . For  $Z_s$  less than  $Z_o$ , effective shunt C has been added and effective series L has been removed. Since these two changes occur at the same position on the  $Z_o$  line, their combined reflection must have additive results. We recall now the required polarity of  $\rho$ , i.e., shunt C has a negative reflection.

$$\rho_c = - \frac{1' Z_o}{2 V_p T_r} \left( \frac{1}{Z_s} - \frac{1}{Z_o} \right)$$

then:

$$\begin{aligned} \rho_{obs} &= \rho_c + \rho_L = \\ &= \frac{1'}{2 V_p T_r} \left[ \frac{Z_s - Z_o}{Z_o} + Z_o \left( \frac{1}{Z_o} - \frac{1}{Z_s} \right) \right] \end{aligned}$$

or:

$$\rho_{obs} = \frac{2 1'}{V_p T_r} \left( \frac{Z_s^2 - Z_o^2}{4 Z_o Z_s} \right)$$

In a transmission line distance, transit time and velocity of propagation have the relationship:

$$l' = v_p T$$

or:

$$\frac{l'}{v_p} = T$$

Thus,  $\rho_{obs}$  becomes:

$$\rho_{obs} = \frac{2 T}{T_r} \left( \frac{Z_s^2 - Z_o^2}{4 Z_o Z_s} \right)$$

Now, to show that Lewis & Wells' approximation is equivalent to the above:

Equation 1

$$\rho_{obs} = \frac{2 T}{T_r} \rho_{act} = \frac{2 T}{T_r} \left( \frac{Z_s - Z_o}{Z_s + Z_o} \right)$$

which can be transformed to:

Equation A

$$\rho_{obs} = \frac{2 T}{T_r} \left( \frac{Z_s^2 - Z_o^2}{Z_s^2 + 2 Z_o Z_s + Z_o^2} \right)$$

Let us assume that  $Z_s - Z_o = E$ , where  $E$  is small compared to  $Z_s$ :

$$Z_s - Z_o = E$$

$$Z_s^2 - 2 Z_s Z_o + Z_o^2 = E^2$$

$$(Z_s + Z_o)^2 = E^2 + 4 Z_s Z_o$$

then:

$$\frac{(Z_s + Z_o)^2}{4 Z_o Z_s} = 1 + \frac{E^2}{4 Z_s Z_o}$$

The approximation is good to 1/4 of a second order error.

The above transformation Equation A finally becomes:

$$\rho_{obs} = \frac{2 T}{T_r} \left( \frac{Z_s^2 - Z_o^2}{4 Z_o Z_s} \right)$$

Thus, Sugarman's approximation has been transformed to Lewis & Wells' relationship.

x x x x x x x x x

References:

- (1) Lewis & Wells, "Millimicrosecond Pulse Techniques," Second Edition, Pergamon Press, 1955, p. 31.
- (2) Sugarman, R. & Merritt, F., "Reflection Coefficient of 125-ohm Amphenol Connectors," Brookhaven National Laboratory, March 1958.

## List of Suggested Equipment:

### Sampling Oscilloscope

Tektronix type	4S1/5T1/661	or
	X4S2/5T1/661	or
	3S76/3T77/561	

### \* Pulse Generator

Tektronix type	110	or
	109	with Trigger Takeoff Transformer (TTO)

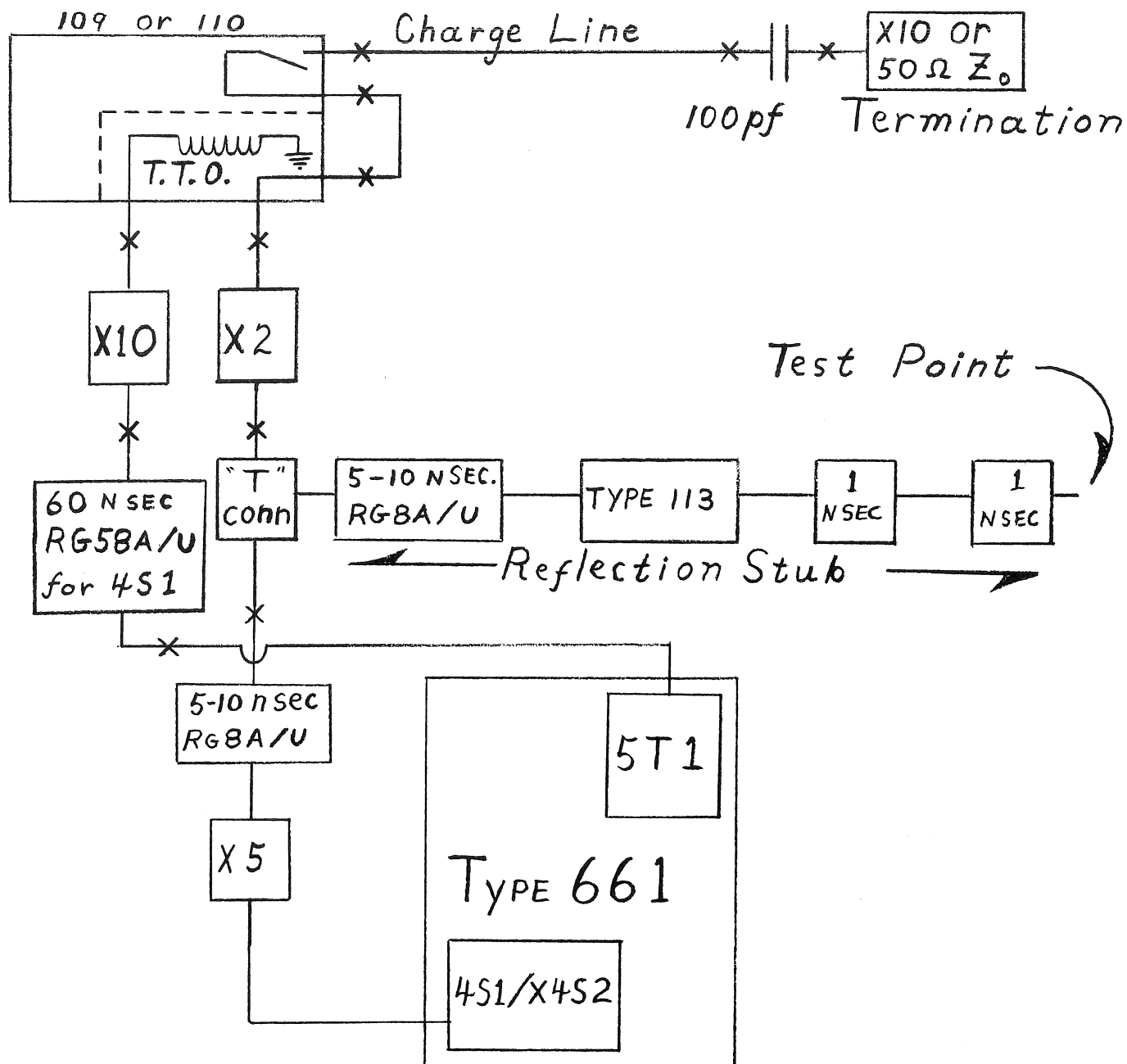
### Coax Cables

Tektronix type	113	(50-ohm, 0.1 nSec $T_r$ )
RG - 58A/U		60 nSec in length
General Radio type GR 874-L30 Air line		(50-ohm, 1 nSec)
		1 or 2
RG - 8A/U	Charge line	5 to 10 nSec
RG - 8A/U	Connecting cables	

### Miscellaneous

General Radio type GR 874-T	(50-ohm "T")
Coupling Capacitor	(50-ohm, 100 to 125 pf)
Attenuators	

\* High repetition rate pulser with 200 pSec  $T_r$  or better is desirable if available.



Block diagram of basic system with type 110 or 109 as pulse source.

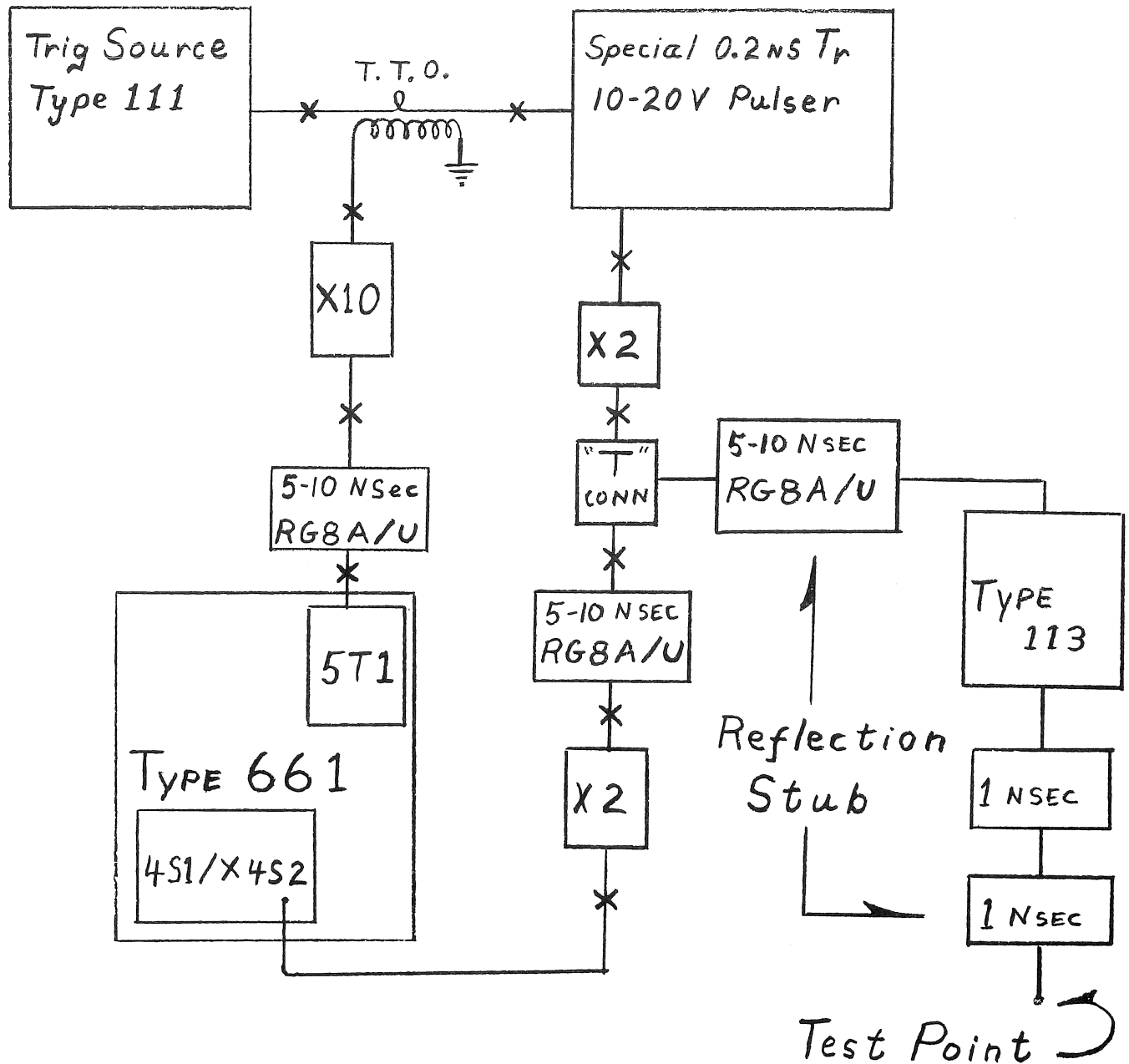


Figure 2

Block diagram of system with high repetition rate pulser.



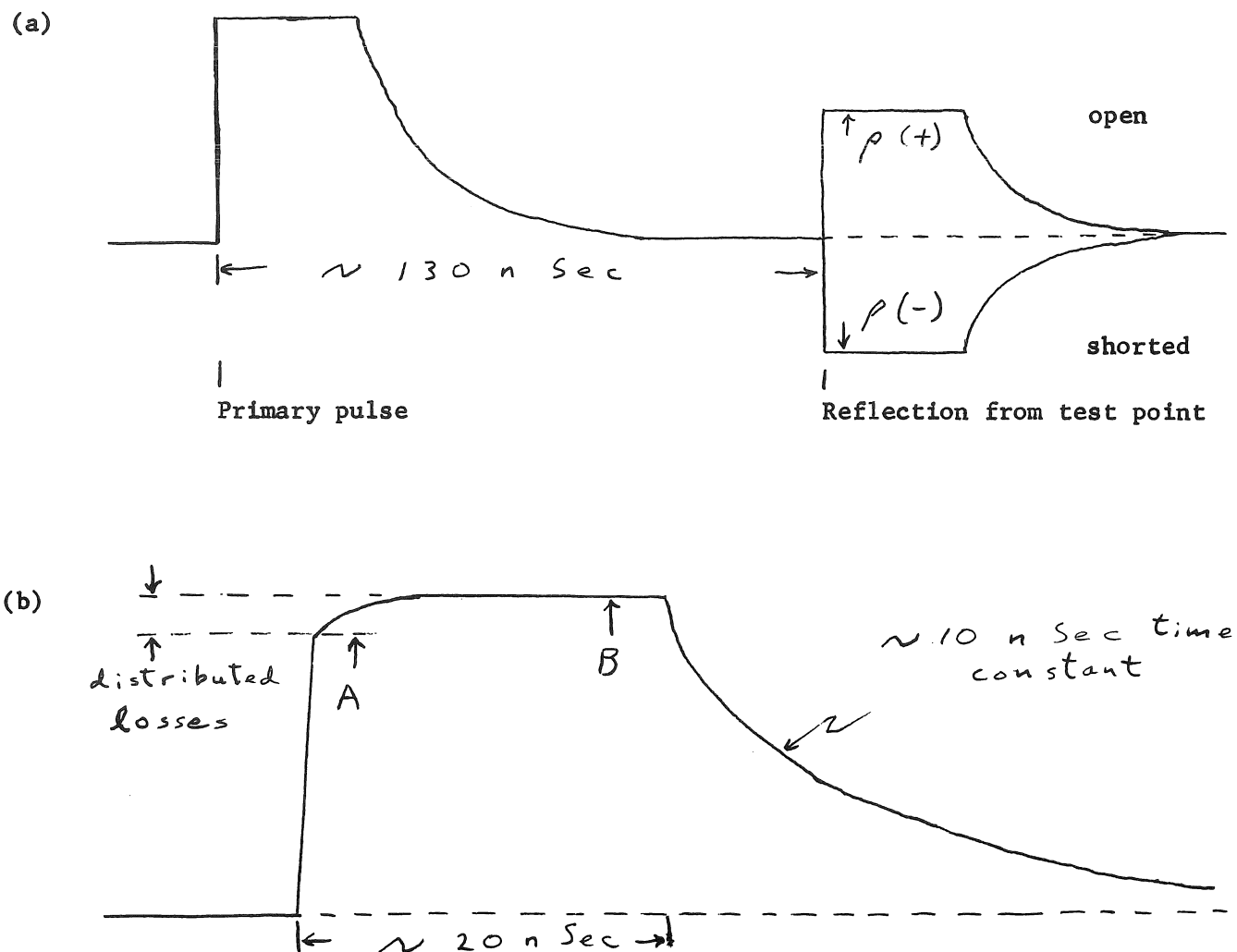


Figure 3

- (a) Line drawing of complete pulse train as displayed by the vertical sampling unit.
- (b) Details of reflection from test point (open circuited).

discontinuity



discontinuity  
added ahead  
of original



discontinuity  
positioned  
after original



correct position,  
wrong magnitude



Figure 4

Determination of position of a single discontinuity.

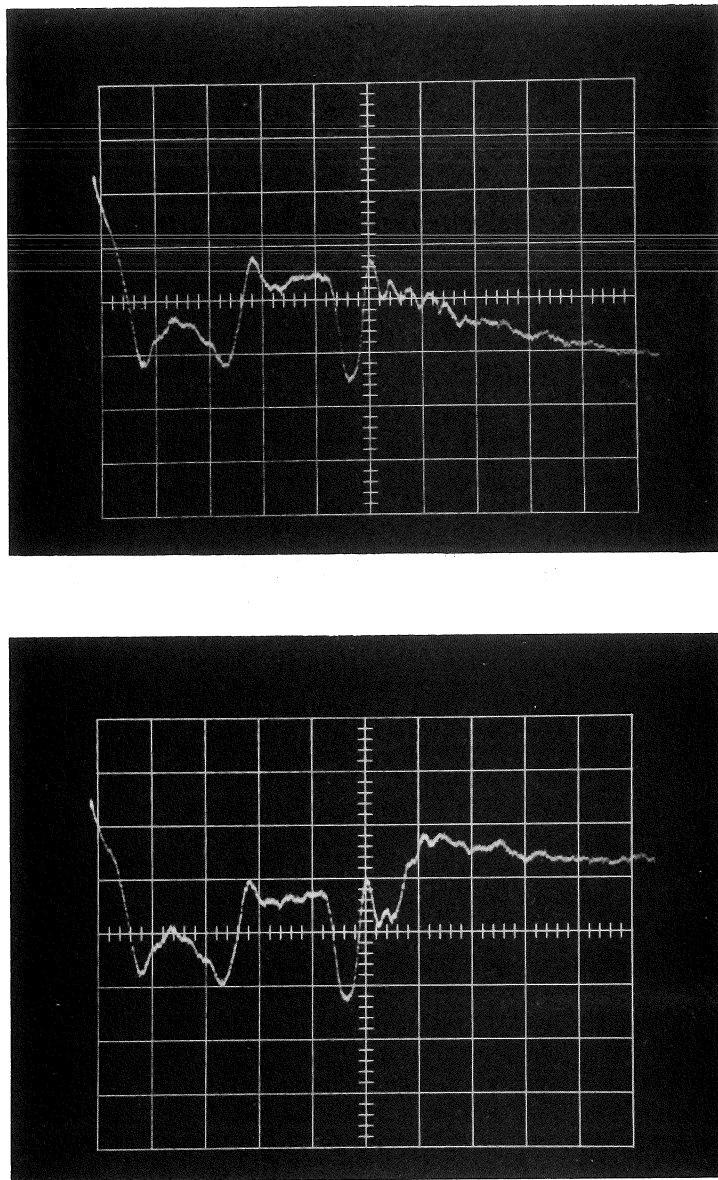


Figure 5

Displays using Type 4S1 reflection system with Type 110 pulser. Sweep rate 1 nSec/cm. Vertical sensitivity  $\sim 2$  mv/cm. Reflection sensitivity is 0.1% per vertical centimeter with system risetime of 400 pSec. The left half of the photos show reflections from the two sections of 50-ohm air line ahead of the test point. The right half illustrates an impedance measurement of a resistive termination. Upper photo terminates reflection stub in nominal 50-ohm (two X10 attenuators). The lower photo has a termination which is approximately  $1/4$  ohm larger (a X10 followed by a X2 attenuator). The discontinuity just before the middle of each photo represents an equivalent capacitance of  $0.04$  pf and is caused by a small hex nut on the center conductor inside the first X10 attenuator.

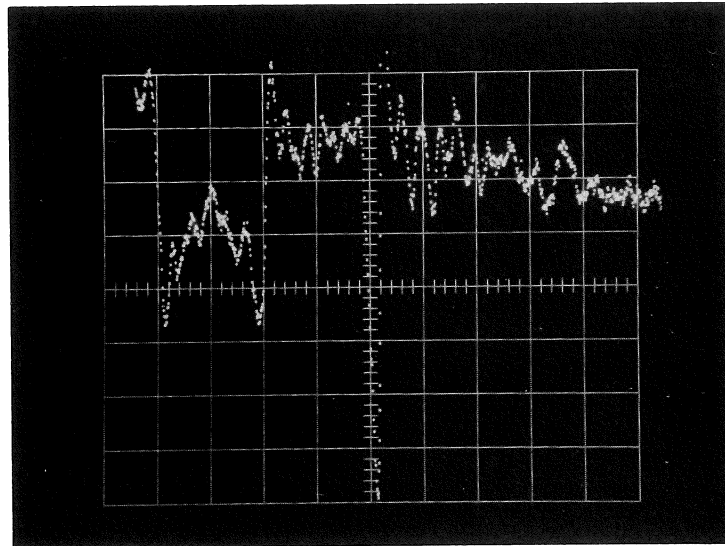


Figure 6

Display of same resistive termination and capacitive discontinuity as shown in Figure 5, upper photo, using Type X4S2 system. Sweep rate 1 nSec/cm.

Reflection resolution of 0.1% per centimeter. System risetime with Type 110 pulser is approximately 200 pSec. With this system, an observed reflection of 0.05% (1/2 centimeter on above display) is caused by an equivalent shunt capacitance of 0.004 pf.

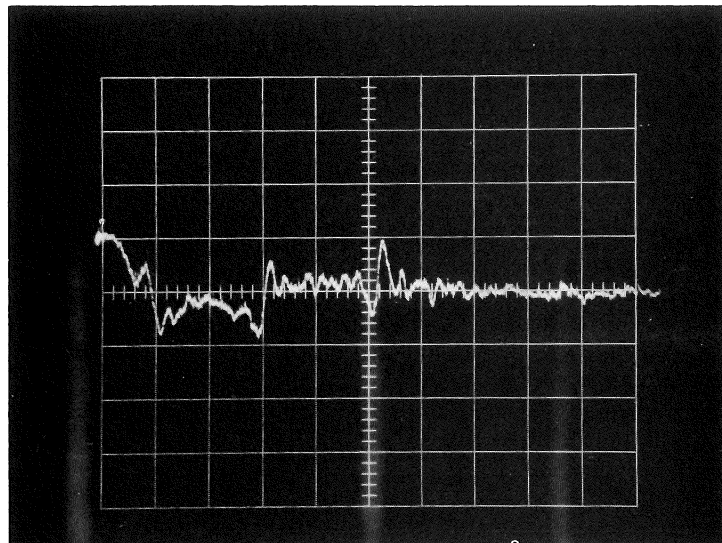
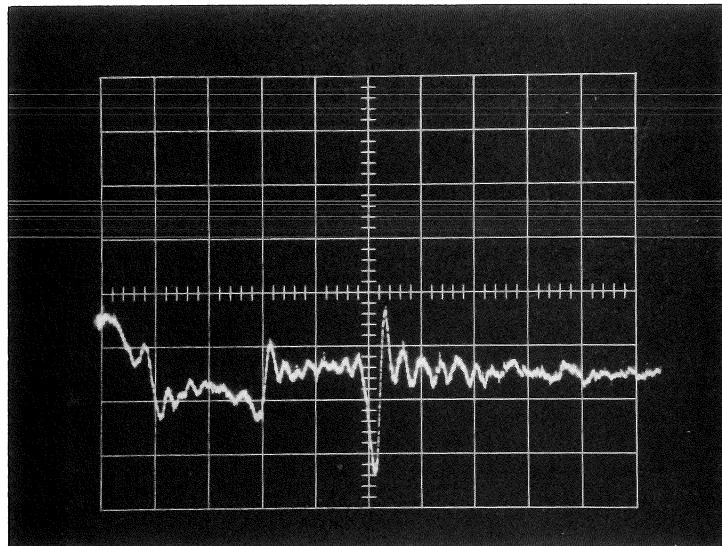


Figure 7

Displays using Type X4S2 system with Type 110 pulser. Sweep rate 1 nSec/cm. Reflection resolution is 0.4% per centimeter. Upper photo is the same as Figure 6, but at reduced sensitivity. The capacitive discontinuity (0.04 pf) is caused by a small hex nut on the center conductor of the first X10 attenuator terminating the reflection stub. The lower photo shows the reflection with the hex removed. Note that there is still a series inductance further inside the attenuator (about 1.5 centimeters away).