TEKTRONIX

SCIENTIST 909
Calculator

WORKBOOK

070-1269-00 1071

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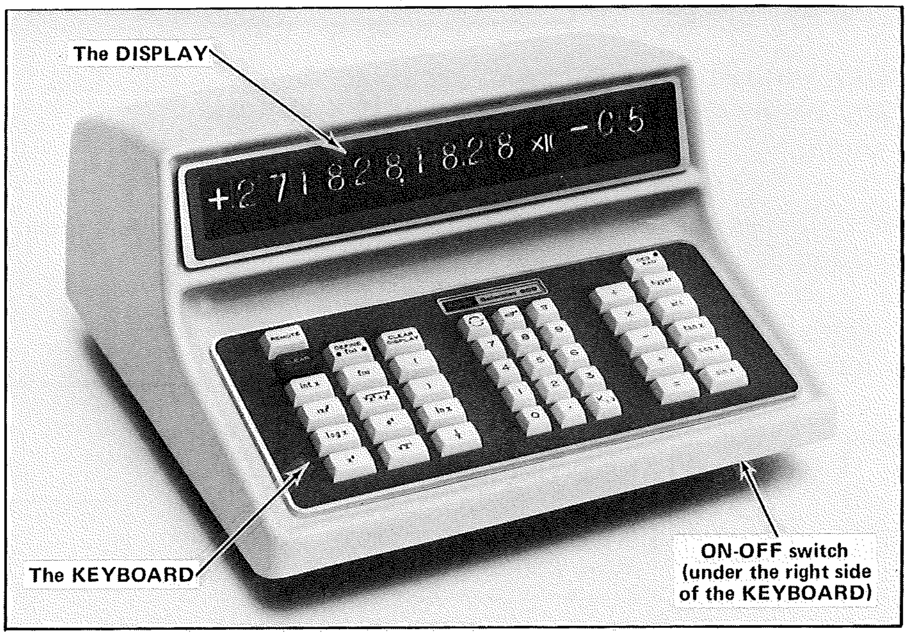
GETTING STARTED

The TEKTRONIX SCIENTIST 909 is a friendly machine. It looks like a calculator. It is, but it is also much more. The SCIENTIST is a computer, capable of working on a problem under control of a stored program. Let's call it a programmable calculator.

Here is the SCIENTIST



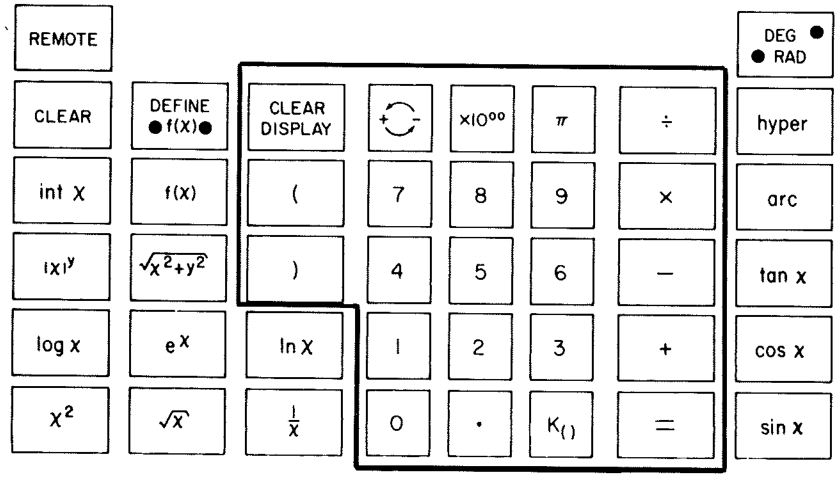
You will use the SCIENTIST first as a calculator, later as a computer. In either case,

- You do the thinking.
- You press the keys.
- The SCIENTIST does the tedious work of numerical calculation.

Let's begin. If the SCIENTIST is not already turned ON, turn it ON. The display will light up, full of zeros, and will blink . . . blink . . . blink at you. Press the red key.

CLEAR

The TEKTRONIX SCIENTIST is now ready to follow your every command.



In the beginning, we will use only these keys and the CLEAR key. The CLEAR key resets the machine . . . it "clears" the deck for action.

Our first problems are easy. You should be able to check the answers by fast mental arithmetic.

DO THESE. Press the keys in left to right order and record the answer that appears in the display.

7 + 5 = ?	KEYS: CLEAR	7	+	5	=		______
7 - 5 = ?	KEYS: CLEAR	7	−	5	=		______
7 x 5 = ?	KEYS: CLEAR	7	×	5	=		______
7 ÷ 5 = ?	KEYS: CLEAR	7	÷	5	=		______
1 ÷ 0 = ?	KEYS: CLEAR	1	÷	0	=		TROUBLE !

The blinking 9's indicate that you have done an illegal operation . . . you can't divide by zero. To remove the error condition, press

CLEAR

MORE THAN ONE OPERATION? OF COURSE.

2 + 3 + 4 =	CLEAR	2	+	3	+	4	=	9
2 x 3 x 4 =	CLEAR	2	×	3	×	4	=	24
2 ÷ 3 ÷ 4 =	CLEAR	2	÷	3	÷	4	=	.1666666667

NEXT ? MIXED OPERATIONS.

2 x 3 + 4 =	2 + 3 x 4 =	2 + 3 ÷ 4 =
CLEAR	CLEAR	CLEAR
2	2	2
×	+	+
3	3	3
+	×	÷
4	4	4
= 10	= 14	= 2.75

2 x 3 + 4 x 5 =									
CLEAR	2	×	3	+	4	×	5	=	26

In mathematics, we encounter expressions such as

$$2(3 + 4) \qquad \frac{2}{3 + 4} \qquad (2 + 3)(4 + 5)$$

Tell it to the SCIENTIST like this:

$2(3 + 4) =$	CLEAR	2	×	(	3	+	4	)	=	14
$\frac{2}{3 + 4} =$	CLEAR	2	÷	(	3	+	4	)	=	.2857142857

But note . .

CLEAR	2	(	3	+	4	)	=

INCORRECT

Your turn . . . fill in the keys for (2 + 3) (4 + 5).

CLEAR													45

Then compute (using the SCIENTIST) $\frac{2 + 3}{4 + 5}$ = ________________

Guess what we use the [.] key for? Try this one.

CLEAR	1	.	2	3	×	4	.	5	6	7	=	________

Did you get 5.61741? We did.

Can I do this? Can I do that? What will the SCIENTIST do if I

____?

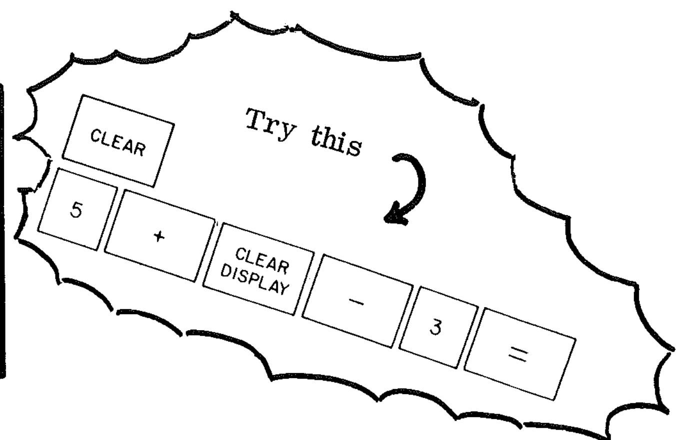
(You complete the question.)

Obviously, we cannot answer all your questions in this workbook. But you and the TEKTRONIX SCIENTIST can answer most of them.

EXPERIMENT! GAMBLE! GUESS, THEN TRY IT!

Do you occasionally make mistakes?

If you make a mistake while entering a number, simply press CLEAR DISPLAY and enter the number correctly.



The CLEAR DISPLAY key clears only the number in the display. It does not reset the machine as does the CLEAR key.

Your turn . . . use the SCIENTIST

(1) Grocery list.

\$1.98
.95
2.37
.59
1.66
?

(2) $5 \times 9 \div 7 =$ ____________

(3) $\dfrac{(2+3)\,(4+5)}{(6+7)} =$ ____________

NEGATIVE NUMBERS

My math book tells me that

(-3) x 4 = -12   3 x (-4) = -12   (-3) x (-4) = 12

But does the SCIENTIST know? Let's find out.

CLEAR	(	3	+/-	)	×	4	=	-12

Enter -3 like this

You have probably guessed that pressing the [+/-] key causes the SCIENTIST to change the sign of the number in the DISPLAY from + to - or from - to +.

Another way - try it!

CLEAR	3	+/-	×	4	=	-12

-3 x 4 = -12

3 x (-4) = ?

CLEAR	3	×	4	+/-	=	-12

-4

Your turn - fill in the keys below, then do it on the calculator and write in the answer.

(-3) x (-4) =   [ ] [ ] [ ] [ ] [ ] [ ] [ ] ________

VERY LARGE NUMBERS

The Gross National Product (GNP) of the U.S. is about 1 trillion dollars.

$$1 \text{ Trillion} = 1,000,000,000,000 = 1 \times 10^{12}$$

The population of the U.S. is about 200 million people.

$$200 \text{ Million} = 200,000,000 = 200 \times 10^{6}$$

If we divide the GNP by the population, we get the GNP per person or GNP per capita.

Lazy? Error prone? Use the calculator!

First, find the $\boxed{\times 10^{00}}$ key (above the $\boxed{8}$ key).

STEP 1. Press $\boxed{\text{CLEAR}}$

STEP 2. Enter GNP $\boxed{1}\,\boxed{\times 10^{00}}\,\boxed{1}\,\boxed{2}$ (1 trillion)

STEP 3. Press $\boxed{\div}$

STEP 4. Enter population $\boxed{2}\,\boxed{0}\,\boxed{0}\,\boxed{\times 10^{00}}\,\boxed{6}$ (200 million)

STEP 5. Press $\boxed{=}$

The answer in the display is $+5.000000000 \times 10^{+03}$

In every day notation, the result is $5000 per person.

$$5.000000000 \times 10^{+03} = 5000$$

This is called the mantissa.

This is called the exponent . . . it tells you to move the decimal point of the mantissa three places to the right.

The land area of the U.S. is about 3.55×10^{6} square miles.

How many people per square mile? Simple - divide the number of people (200 million) by the number of square miles (3.55×10^{6}).

CLEAR	2	0	0	$\times 10^{00}$	6	$\div$	3	.	5	5	$\times 10^{00}$	6	=

$5.633802817 \times 10^{+01}$

Let's change that answer to everyday notation.

$$5.633802817 \times 10^{+01} = 56.33802817$$

In other words, about 56 people per square mile.

Turn the problem around. How many square miles per person?

CLEAR	3	.	5	5	$\times 10^{00}$	6	$\div$	2	0	0	$\times 10^{00}$	6	=

1.775×10^{-02}

This time, the exponent is negative. It tells us to move the decimal point of the mantissa to the left.

$1.775 \times 10^{-02} = .01775$ square miles per person.

We had to add a zero

What is 1.2345×10^{-3} in everyday notation? ______________________

What is 1.2345×10^{7} in everyday notation? ______________________

THE CROWDED EARTH

TOO MANY PEOPLE

The population of the earth (1970) is about 3.5 billion people.

$$3.5 \text{ Billion} = 3.5 \times 10^9$$

If the present growth rate persists, the population will <u>double</u> every 35 years. In this event, what will the population be in 350 years (in the year 2320)?

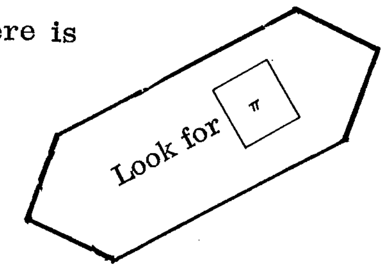
CLEAR		
3 . 5 x10^{00} 9	Start with 3.5 billion people in 1970,	
× 2	double it	(year 2005)
× 2	and double that	(year 2040)
× 2	and double again	(year 2075)
× 2	and again	(year 2110)
× 2	and again	(year 2145)
× 2	and	
× 2	so	
× 2	on	
× 2	until we reach	
× 2	the year 2320	
= 3.584×10^{12}	The answer is . . . too many people!	

All those people must have a place to live. Let's suppose they live on the land surface of the earth. What is the land surface of the earth? I don't know - but I do know this:

- The earth is approximately a sphere in shape.
- Its radius is about 4000 miles.
- The surface area S of a sphere is

$$S = 4\pi r^2$$

where r is the radius.



Let the calculator do it!

$$\boxed{\text{CLEAR}}\ \boxed{4}\ \boxed{\times}\ \boxed{\pi}\ \boxed{\times}\ \underbrace{\boxed{4}\ \boxed{0}\ \boxed{0}\ \boxed{0}\ \boxed{\times}\ \boxed{4}\ \boxed{0}\ \boxed{0}\ \boxed{0}}_{r^2 = r \times r}\ \boxed{=}$$

Did you get 201061929.8 square miles? Hope so.

Let's sneak in a new key. This one: $\boxed{x^2}$

$$\boxed{\text{CLEAR}}\ \boxed{4}\ \boxed{\times}\ \boxed{\pi}\ \boxed{\times}\ \underbrace{\boxed{4}\ \boxed{0}\ \boxed{0}\ \boxed{0}\ \boxed{x^2}}_{4000^2}\ \boxed{=}$$

Did you get 201061929.8 square miles?

Now you can compute the number of people per square mile. But wait! I computed the total surface area of the earth, land and sea. The land area is about 29% of the total area (I think).

What is the land area of the earth? ______________________

Let's compute the number of people per square mile of earth surface, land and sea, in 1970.

CLEAR

3 · 5 ×10⁰⁰ 9

÷

2 0 1 0 6 1 9 2 9 · 8

= $1.740757190 \times 10^{+01} \cong$ 17 people/Sq. Mi.

"approximately equal to"

Because $1.740757190 \times 10^{+01} = 17.40757190 \cong 17$ ("nearest person")

Since we are satisfied with an approximate result, we could have done it like this.

CLEAR

3 · 5 ×10⁰⁰ 9

÷

2 · 0 1 ×10⁰⁰ 8

= $1.741293532 \times 10^{+01} \cong$ 17

"approximately equal to"

Because $\underbrace{201061929.8}_{\text{8 places}} = 2.010619298 \times 10^{8} \cong 2.01 \times 10^{8}$

check the exponent over here and here

Your turn. On Page 10, did you get 58307959.64 miles for the land surface of the earth? If you didn't, try again. In any event, use our result for the following.

<u>Question</u> For 1970, how many people per square mile of land surface on the earth? ____________

<u>Question</u> For the year 2320, how many people per square mile on the surface of the earth, land and sea? ________________________

Land surface only? ____________________

Question of great importance to your descendants.

If the population continues to double every 35 years, how many people will there be 700 years from 1970 (in the year 2670)?

Your answer: ______________________

How many people in 3020 (1050 years from 1970)?

Your answer: ______________________

How many people per square mile of earth surface, land and sea, in 3020?

Your answer: _____________________

How many people per <u>square yard</u>?! (There are 1760 yards in one mile . . . 1760^2 square yards in a square mile.)

Your answer: _____________ people per square yard.

COMPUTER POWER

Remember how we computed the predicted world population in the year 2320 (assuming the population doubles every 35 years).

$\boxed{\text{CLEAR}}\ \boxed{3}\ \boxed{\cdot}\ \boxed{5}\ \boxed{\times 10^{00}}\ \boxed{9}\ \underbrace{\boxed{\times}\ \boxed{2}\ \boxed{\times}\ \boxed{2}\ \cdots\ \boxed{\times}\ \boxed{2}}_{\boxed{\times}\ \boxed{2}\ \text{ten times . . .}}\ \boxed{=}$

In other words, the population in 2320 is

$$(3.5 \times 10^{9}) \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

but $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ is 2^{10}.

Look for $\boxed{|x|^y}$ Use it, below.

$\underbrace{\boxed{\text{CLEAR}}\ \boxed{3}\ \boxed{\cdot}\ \boxed{5}\ \boxed{\times 10^{00}}\ \boxed{9}}_{\text{1970 population}}\ \underset{\times}{\boxed{\times}}\ \underbrace{\boxed{2}\ \boxed{|x|^y}\ \boxed{1}\ \boxed{0}}_{2^{10}}\ \boxed{=}\ \ = 3.584 \times 10^{+12}$

But what if the population continues to double every 35 years (dreadful thought!). What will it be in the year 2670.

$$2670 - 1970 = 700 = 20 \times 35 \quad (\text{20 doublings})$$

$\boxed{\text{CLEAR}}\ \boxed{3}\ \boxed{\cdot}\ \boxed{5}\ \boxed{\times 10^{00}}\ \boxed{9}\ \boxed{\times}\ \boxed{2}\ \boxed{|x|^y}\ \boxed{2}\ \boxed{0}\ \boxed{=}\ \ 3.670015999 \times 10^{+15}$

But let's just call it 3.67×10^{15} people.

Your turn . . . what will the population be in the year 3020?

____

My World Almanac tells me that the mass of the earth is six sextillion, 588 quintillion tons.

6,588,000,000,000,000,000,000 tons.

We can also write it as

$$6588 \times 10^{18} \quad \text{or} \quad 6.588 \times 10^{21} \quad \text{short tons.}$$

One ton is 2000 pounds. The mass of the earth is

CLEAR	2	0	0	0	×	6	5	8	8	$\times 10^{00}$	1	8	=

<u>1.3176×10^{25} pounds</u>

Heavy! But let's check it out. The earth is nearly a sphere and the <u>volume</u> of a sphere is

$$V = \frac{4}{3}\pi r^3$$

where <u>r</u> is the radius. The radius of the earth is about 4000 miles. So the volume of the earth is about

CLEAR	4	÷	3	×	π	×	4	0	0	0	$\lvert x \rvert^y$	3	=

2.681×10^{11} cubic miles

Let's change it to cubic feet.

CLEAR	2	.	6	8	1	$\times 10^{00}$	1	1	×	5	2	8	0	$\lvert x \rvert^y$	3	=

$$\text{(cubic miles)} \times \text{(feet per mile)}^3 \cong 3.946 \times 10^{22}$$

The mass of one cubic foot of earth, on the average, is 5.52 times the mass of one cubic foot of water. The mass of one cubic foot of water is 62.4 pounds. The mass of the earth?

CLEAR	3	.	9	4	6	×10ºº	2	2	×	5	.	5	2

×	6	2	.	4	=

about $\underline{1.359 \times 10^{25}}$ pounds.

WORLD ALMANAC	ME AND THE SCIENTIST
1.3176×10^{25} lbs.	1.359×10^{25} lbs.

Not bad! We are a little high. Why? For one thing, the earth is not quite a sphere. The equatorial diameter is 7908 miles and the polar diameter is 7882 miles. Therefore,

Equatorial radius = 3959 miles
Polar radius = 3941 miles

Average radius = (3959 + 3941) ÷ 2 = 3950 miles

Your turn . . . redo our calculations using 3950 miles as the radius of the earth. What do you get for the mass of earth?

YOU AND THE SCIENTIST
________________ pounds

Now convert your answer to tons. ____________________ tons.

Try these.

Enter 12345 [CLEAR] [1] [2] [3] [4] [5]

Then press [$\times 10^{00}$] [=] $+1.23450000 \times 10^{+04}$

You have converted 12345 to <u>scientific notation</u>.

Now find the [DEG ● / ● RAD] key. Press it twice.

The display should now read +0000012345.

You have converted $+1.234500000 \times 10^{+04}$ back to standard notation. More examples follow.

[CLEAR] [.] [0] [0] [1] [2] [3] [$\times 10^{00}$] [=] $+1.230000000 \times 10^{-03}$

<u>DON'T</u> CLEAR - press [DEG ● / ● RAD] [DEG ● / ● RAD] +000000.00123

Try this <u>program</u> (sequence of steps) for each of these numbers.

STEP 1. [CLEAR]

STEP 2. Enter the number.

STEP 3. [$\times 10^{00}$] [=]

STEP 4. Read the display.

STEP 5. [DEG ● / ● RAD] [DEG ● / ● RAD]

STEP 6. Read the display.

.1
.01
.001
.0001

10
100
1000
10000

Do you see a pattern?

PARTING SHOT. If a snail slithers one inch per second, how long will it take the snail to slither completely around the earth following the equator? (Assume that the snail can slither on water.)

Your answer ____________ seconds ____________ days ____________ years

INTERLUDE

FUNCTION MACHINE

On the left side of the keyboard, find the following keys.

$\boxed{x^2}$ $\boxed{\sqrt{x}}$ $\boxed{\frac{1}{x}}$

Each of these keys tells the SCIENTIST to compute a <u>function</u> of the number in the display and return the result to the display. We have already used the $\boxed{x^2}$ key, so let's recall what it does.

$\boxed{x^2}$ compute the square of the number in the display and put the result into the display.

Do it!

$\underbrace{\boxed{\text{CLEAR}}\ \boxed{5}\ \boxed{x^2}}$ 25 — 5^2 is 25

$\underbrace{\boxed{\text{CLEAR}}\ \boxed{2}\ \boxed{3}\ \boxed{x^2}}$ 529 — 23^2 is 529

Your turn:

	$\boxed{\text{CLEAR}}\ \boxed{1}\ \boxed{\cdot}\ \boxed{2}\ \boxed{3}\ \boxed{x^2}$	______
$(-3)^2 =$	$\boxed{\text{CLEAR}}\ \boxed{3}\ \boxed{+\circlearrowleft-}\ \boxed{x^2}$	9
$(2 \times 10^{49})^2 =$	$\boxed{\text{CLEAR}}\ \boxed{2}\ \boxed{\times 10^{\circ\circ}}\ \boxed{4}\ \boxed{9}\ \boxed{x^2}$	$+4.000000000 \times 10^{+98}$
$(2 \times 10^{50})^2 =$	$\boxed{\text{CLEAR}}\ \boxed{2}\ \boxed{\times 10^{\circ\circ}}\ \boxed{5}\ \boxed{0}\ \boxed{x^2}$	OVERFLOW !

OVERFLOW occurs because $(2 \times 10^{50})^2 = 4 \times 10^{100}$ is too big. The SCIENTIST signals that some problem has occurred by blinking.

$$\boxed{+9.999999999 \times 10^{+99}}$$

Simply press $\boxed{\text{CLEAR}}$ and do another problem.

Try $(2 \times 10^{-50})^2$

Let's check out a new key. This one . . . $\boxed{\frac{1}{x}}$

$\boxed{\frac{1}{x}}$ compute the <u>reciprocal</u> of the number in the display and return the result to the display.

Try it!

$\boxed{\text{CLEAR}}\ \boxed{2}\ \boxed{\frac{1}{x}}$ .5

Exactly

$\boxed{\text{CLEAR}}\ \boxed{3}\ \boxed{\frac{1}{x}}$ .3333333333

Well, almost

Looking for trouble? Try this $\boxed{\text{CLEAR}}\ \boxed{\frac{1}{x}}$

The number 0 does not have a reciprocal . . . $\frac{1}{0}$ is not defined.

Let's use it.

1 foot = 0.3048 meter

$\boxed{\text{CLEAR}}\ \boxed{\cdot}\ \boxed{3}\ \boxed{0}\ \boxed{4}\ \boxed{8}\ \boxed{\frac{1}{x}}$ 3.280839895

1 meter = 3.280839895 feet $\cong$ 3.281 feet

1 pound, avoirdupois = 453.59237 grams

$\boxed{\text{CLEAR}}\ \boxed{4}\ \boxed{5}\ \boxed{3}\ \boxed{\cdot}\ \boxed{5}\ \boxed{9}\ \boxed{2}\ \boxed{3}\ \boxed{7}\ \boxed{\frac{1}{x}}$

$2.204622622 \times 10^{-03}$

1 gram = $2.204622622 \times 10^{-03} \cong 0.002205$ lbs.

YOUR TURN

1 gallon (U.S.) = 3.785 liters

1 liter = ____________________ gallon (U.S.)

How many liters in a fifth of a gallon?

CLEAR 5 $\frac{1}{x}$ × 3 . 7 8 5 = .757

Of course, you could have done it like this

CLEAR 3 . 7 8 5 ÷ 5 = .757

But, after all, we are discussing the $\frac{1}{x}$ key.

How many liters in a quart?

CLEAR 4 $\frac{1}{x}$ × 3 . 7 8 5 = .94625

A liter is very nearly a quart. In fact, one liter is (if you haven't pressed CLEAR or CLEAR DISPLAY)

$\frac{1}{x}$ 1.05680317 QUARTS

But if you had already pressed CLEAR or CLEAR DISPLAY

CLEAR . 9 4 6 2 5 $\frac{1}{x}$ 1.05680317

Computing power! Try these.

$$\boxed{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = ?}$$

CLEAR 2 $\frac{1}{x}$ + 3 $\frac{1}{x}$ + 4 $\frac{1}{x}$ = 1.083333333

$$\boxed{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = ?}$$

CLEAR 1 − 2 $\frac{1}{x}$ + 3 $\frac{1}{x}$ − 4 $\frac{1}{x}$ =

.5833333333

Suppose you wanted to compute $\frac{1}{x^2}$ for some number x.

$1/5^2 =$	CLEAR	5	x^2	$\frac{1}{x}$	.04
$1/5^2 =$	CLEAR	5	$\frac{1}{x}$	x^2	.04

Because $\frac{1}{x^2} = \left(\frac{1}{x}\right)^2$

$$1 + \frac{1}{2^2} + \frac{1}{3^2} = ?$$

CLEAR	1	+	2	x^2	$\frac{1}{x}$	+	3	x^2	$\frac{1}{x}$	=	1.361111111

Your turn - check our work - right or wrong?

1 - 1/2 + 1/3 - 1/4 + 1/5 - 1/6 + 1/7 - 1/8 + 1/9 = .7456349206

$1 + 1/2^2 + 1/3^2 + 1/4^2 + 1/5^2 + 1/6^2 + 1/7^2 + 1/8^2 + 1/9^2 =$ 1.539767731

$1/3^2 + 1 =$ 1.111111111

$1/3^2 + 4^2 =$ .0625

Still your turn - fill in the missing keystrokes.

$$\frac{1}{7^2} - \frac{1}{5^2} =$$

CLEAR								=

$-1.959183673 \times 10^{-02}$

$$\frac{1}{7^2 - 5^2} =$$

CLEAR						=	$\frac{1}{x}$

$4.166666667 \times 10^{-02}$

Two ways to compute $\frac{1}{2 + 3}$

CLEAR	2	+	3	=	$\frac{1}{x}$

.2

CLEAR	(	2	+	3	)	$\frac{1}{x}$

.2

Two ways to compute $(3 + 4)^2$

CLEAR	3	+	4	=	x^2

49

CLEAR	(	3	+	4	)	x^2

49

* REMEMBER **

x^2 compute the square of the number that is in the display <u>at the time that you press the key</u>.

$\frac{1}{x}$ compute the reciprocal of the number that is in the display <u>at the time that you press the key</u> .

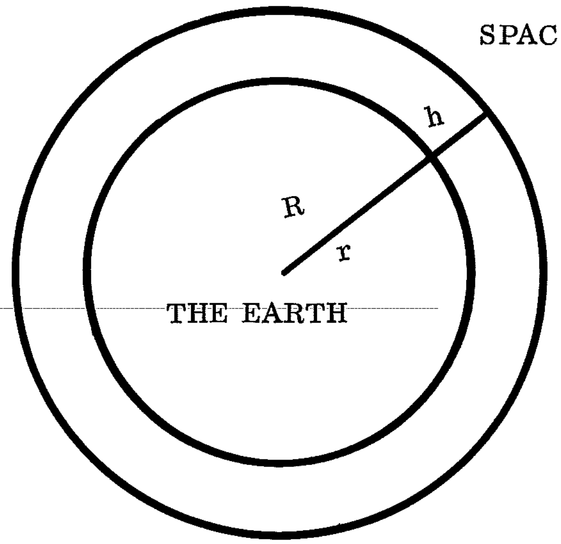
****

Another <u>function key</u> $\boxed{\sqrt{x}}$

$\boxed{\sqrt{x}}$ compute the square root of the number in the display and put the result into the display.

$\boxed{\text{CLEAR}}\ \boxed{9}\ \boxed{\sqrt{x}}$ <u>3</u> $\boxed{\text{CLEAR}}\ \boxed{2}\ \boxed{\sqrt{x}}$ <u>1.414213562</u>

SPACE FLIGHT



News flash! Snoopy is in orbit around the earth at an altitude (h) of 1000 miles.

h = 1000 miles
R = 3950 miles
r = R + h = 4950 miles.

In order to stay in his circular orbit 1000 miles above the earth, Snoopy must maintain a speed of

$$v = \sqrt{\frac{9.56 \times 10^{4}}{r}} \text{ miles per second}$$

What is Snoopy's speed?

$\boxed{\text{CLEAR}}$

$\boxed{9}\ \boxed{.}\ \boxed{5}\ \boxed{6}\ \boxed{\times 10^{00}}\ \boxed{4}$

$\boxed{\div}$

$\boxed{4}\ \boxed{9}\ \boxed{5}\ \boxed{0}$

$\boxed{=}$

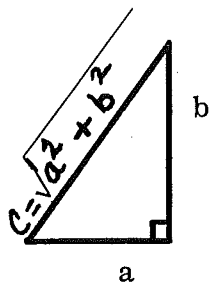
$\boxed{\sqrt{x}}$ <u>$4.394670786 \times 10^{+00}$</u> = 4.39 miles per second

What is Snoopy's speed in miles per hour?

Compute Snoopy's speed if he maintains a circular orbit 2000 miles above the earth's surface (r = 5950)

______ miles/second

PYTHAGORAS



We start with a right triangle.

a = 3
b = 4
c = ________________?

Here is one way to do it.

[CLEAR] [3] [×] [3] [+] [4] [×] [4] [=] [$\sqrt{x}$] ____5____

and another way

[CLEAR] [3] [x^2] [+] [4] [x^2] [=] [$\sqrt{x}$] ____5____

and still another way

[CLEAR] [(] [3] [x^2] [+] [4] [x^2] [)] [$\sqrt{x}$] ____5____

but we recommned this way, using a new key.

[CLEAR] [3] [$\sqrt{x^2+y^2}$] [4] [=] ____5____

Try it . . . complete the table.

a	b	c
12	5	________
5	12	________
1	1	________

MIXED BAG

Pick any problem . . . solve it in at least two <u>different</u> ways. For example, here are three different ways to evaluate

$$\sqrt{p^2 + q^2 + r^2}$$

for p = 3, q = 4 and r = 5.

(1)

CLEAR	3	x^2	+	4	x^2	+	5	x^2	=	$\sqrt{x}$

(2)

CLEAR	3	$\sqrt{x^2+y^2}$	4	=	x^2	+	5	x^2	=	$\sqrt{x}$

(3)

CLEAR	3	$\sqrt{x^2+y^2}$	4	$\sqrt{x^2+y^2}$	5	=

All three methods give the same answer: +7.071067812

PROBLEMS

(1) $\sqrt{\sqrt{\sqrt{2}}}$

(2) $\dfrac{1}{2 + \dfrac{1}{3 + 1/4}}$

(3) $\left(\left(1 + 2^2\right)^2 + 3^2\right)^2$

If you pick one of the following, do it first using our values then do it again using <u>your</u> data.

(4) $\sqrt{\dfrac{1}{a + b}}$ a = 7, b = 5

(5) $\sqrt{\dfrac{1}{a^2 + b^2}}$ a = 6, b = 8

(6) $a^2 + b^2 + c^2$ for a = 1, b = 2, c = 3

(7) $\dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$ for v = 179000, c = 186000

A PROBLEM OF INTEREST

GROWING PAINS

Let's start in "year 0" with a population of P_0 people and assume that the population increases by 1% each year. Let P_1 be the population at the end of year 1, P_2 be the population at the end of year 2, P_3 be the population at the end of year 3 and so on.

What is the population at the end of n years?

Remember - the population increases by 1% each year. Like this. (If you can't follow the algebra - don't worry - simply use the formulas near the bottom of the page.)

$$P_1 = P_0 + P_0(.01) = P_0(1 + .01)$$

$$P_2 = P_1 + P_1(.01) = P_1(1 + .01) = P_0(1 + .01)^2$$

$$P_3 = P_2 + P_2(.01) = P_2(1 + .01) = P_0(1 + .01)^3$$

$$\vdots$$

$$P_n = P_0(1 + .01)^n \longleftarrow \text{n years}$$

percent increase per year

initial population

population at end of n years

If the growth rate is 2% per year, then

$$\boxed{P_n = P_0(1 + .02)^n}$$

and if the growth rate is r% per year,

$$\boxed{P_n = P_0\left(1 + \frac{r}{100}\right)^n}$$

Do it for $P_0 = 1000$ people, $r = 1\%$ and $n = 10$ years.

$$P_{10} = 1000(1 + .01)^{10} = 1000(1.01)^{10}$$

CLEAR	1	0	0	0	×	1	.	0	1	$\|x\|^y$	1	0	=

We got 1104.622125 people.

Let's call it about 1105 people.

<u>You</u> do it for $P_0 = 1000$ people, $r = 1\%$ and $n = 20$ years.
(Show the keys.)

Answer please (to the nearest person)____________________

Another problem for you and the TEKTRONIX SCIENTIST. Do it for $P_0 = 1000$ people, $r = 2\%$ and $n = 40$ years.

Your answer ?__________________________ More than double the initial population!

Once more . . . $P_0 = 3 \times 10^9$ people
$r = 2\%$
$n = 99$ years

Answer ?__________________________

Approximations cheerfully accepted

HOW LONG TO DOUBLE ?

Initial population: P_0

Double it ⟶ $2P_0$

How long does it take at r% increase per year? In other words what is the value of n such that

$$P_n = P_0(1 + \frac{r}{100})^n = 2P_0$$

It must happen when

$$(1 + \frac{r}{100})^n = 2$$

Problem: Given r, find n.

Solution: Use the BRUTE FORCE METHOD with the help of our friendly TEKTRONIX calculator.

Guess a lot. Try out each guess. Think about the result of each guess. Use each result to help make a better guess.

How many years (what is n?) if r = 1%?

Gamble! Try n = 30 years.

`CLEAR` `1` `.` `0` `1` `|x|^y` `3` `0` `=` 1.347848915

Too small (less than 2). Try again . . . n = 100 years.

`CLEAR` `1` `.` `0` `1` `|x|^y` `1` `0` `0` `=` 2.704813829

Too big (more than 2). Let's try n = 80.

`CLEAR` `1` `.` `0` `1` `|x|^y` `8` `0` `=` 2.216715217

Still too big. Try n = 60.

`CLEAR` `1` `.` `0` `1` `|x|^y` `6` `0` `=` 1.816696698

A little too small. Try n = 70.

`CLEAR` `1` `.` `0` `1` `|x|^y` `7` `0` `=` 2.006763368

Right on! But to make sure, try n = 69.

n = 69 ________________________

Your final choice: n = 69 or n = 70?

circle it

THERE IS ALWAYS A BETTER WAY

You and the SCIENTIST have worked diligently solving problems. But you have worked too hard . . . the SCIENTIST can do much more of the work. Find these keys.

[DEFINE ● f(x) ●] [f(x)] [K( )]

Recall your work on Page 28. Think about all those buttons you pressed . . . then try this.

PRESS

[CLEAR]

[DEFINE ● f(x) ●] ← Two small light on the keys (keylights) turn on.

[=] [K( )] [0] [0] [CLEAR DISPLAY] [1] [.] [0] [1] [x^y] [K( )] [0] [0] [=]

[DEFINE ● f(x) ●] ← Lights off

We have <u>defined</u> a <u>function.</u> Now let's <u>use</u> the function.

n = 30	[CLEAR] [3] [0] [f(x)]	<u>1.347848915</u>
n = 100	[CLEAR] [1] [0] [0] [f(x)]	<u>2.704813829</u>
n = 80	[CLEAR] [8] [0] [f(x)]	<u>2.216715217</u>
n = 60	[CLEAR] [6] [0] [f(x)]	<u>1.816696698</u>
n = 70	[CLEAR] [7] [0] [f(x)]	<u>2.006763368</u>

We defined the function

$$f(n) = 1.01^n$$

Then we evaluated the function for five values of n.

- If $n = 30$, then $f(n) = 1.347848915$
- In other words, $f(30) = 1.347848915$
- Using the same type of notation

  $f(100) = 2.704813829$ $\qquad$ $f(80) = 2.216715217$

- Your turn

  $f(60) =$ ______________ $\qquad$ $f(70) =$ __________

If we change the yearly growth rate to 2%, we get a new function.

$$f(n) = 1.02^n$$

Define it for the SCIENTIST

CLEAR

DEFINE ● f(X) ● — Tell it that we want to define a function.

This function

= | K( ) | 0 | 0 | CLEAR DISPLAY | 1 | . | 0 | 2 | $|x|^y$ | K( ) | 0 | 0 | =

DEFINE ● f(X) ● — Tell it that we are finished defining the function

The function $f(n) = 1.02^n$ is now defined. You use it to find the doubling time.

Doubling time is ________ years.

In case you are wondering about

K( ) | 0 | 0

be patient . . . all will be revealed, beginning on Page 33.

But now we have work for you and the SCIENTIST. Complete the following table.

ANNUAL PERCENT INCREASE	DOUBLING TIME (YEARS)
1	70
2	______
3	______
4	______
5	______

Having defined a function, we can use the [f(x)] key in the same way that we used

[x^2] [$\sqrt{x}$] [$\frac{1}{x}$]

For example, let's define the function,

$$f(n) = 1.01^n$$

and then use it to compute the population at the end of n years if we start with an initial population P_0 and have an annual increase of 1%.

$$P_n = P_0 \times 1.01^n = \underbrace{1.01^n}_{\text{Here is } f(n)} \times P_0$$

Let's try it for $P_0 = 1000$ people and several values of n.

First, define the function $f(n) = 1.01^n$.

[CLEAR]

[DEFINE ●f(x)●] ← Lights on

[=] [K( )] [0] [0] [CLEAR DISPLAY] [1] [.] [0] [1] [$|x|^y$] [K( )] [0] [0] [=]

[DEFINE ●f(x)●] ← Lights off

Try it for P_0 = 1000 people and n = 10 years.

[CLEAR] [1] [0] [f(x)] [×] [1] [0] [0] [0] [=]   1104.622125

n

Again for P_0 = 1000 people and n = 20 years.

[CLEAR] [2] [0] [f(x)] [×] [1] [0] [0] [0] [=]   1220.19004

Once more for $P_0 = 3 \times 10^9$ people and n = 70 years.

[CLEAR] [7] [0] [f(x)] [×] [3] [×10⁰⁰] [9] [=] $6.020290104 \times 10^{+09}$

If P_0 = 2000 and n = 50, what is P_n? ____________

The above results hold for r = 1%. If we want to do similar calculations for r = 2%, we define an appropriate function for the SCIENTIST. This one

$$f(n) = 1.02^n$$

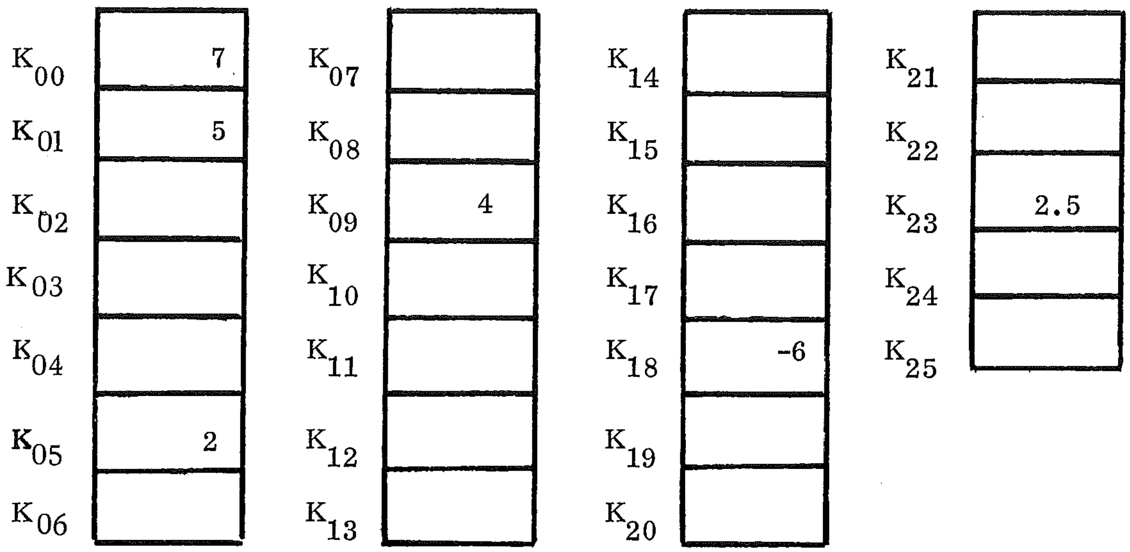
Do it, then complete the following table. We want P_n give to the nearest person.

P_0	n	P_n
1000	10	____________
1000	20	____________
3×10^9	35	____________
3×10^9	70	____________
3×10^9	99	____________

THE MYSTERIOUS MR $K_{(\,)}$

LITTLE BOXES

Deep down inside the SCIENTIST are 26 little boxes. Each box has a name, a label. Here they are.



26 boxes labelled K_{00} through K_{25}

Each box can contain one number at any one time. We have already stored numbers in some of the boxes.

We put 7 in box K_{00}

We put 5 in box K_{01}

What number is in K_{05}? ______ in K_{09}? ______

-6 is in box ______ and 2.5 is in box ______

Your turn. Take pencil in hand and

$8 = K_{07}$ (put 8 into box K_{07})

$7.3 = K_{25}$ (put 7.3 into box K_{25})

Let's store numbers in boxes in the TEKTRONIX SCIENTIST's <u>memory</u>.

- Put 7 into K_{00}

[CLEAR DISPLAY] [7] [=] [K( )] [0] [0] done!

- Put 5 into K_{01}

[CLEAR DISPLAY] [5] [=] [K( )] [0] [1] done!

- Put -6 into K_{18}

[CLEAR DISPLAY] [6] [+ -] [=] [K( )] [1] [8] done!

- Put 2.5 into K_{23}

[CLEAR DISPLAY] [2] [.] [5] [=] [K( )] [2] [3] done!

Then let's read them out (into the display). You press the keys and record the result (in the display).

[K( )] [0] [0] ____________ [K( )] [0] [1] ____________

[K( )] [1] [8] ____________ [K( )] [2] [3] ____________

The numbers are still stored in K_{00}, K_{01}, K_{18} and K_{23}. The readout process does not destroy them. To convince yourself of this fact, read them out again.

TO STORE A NUMBER

(1) Press [CLEAR DISPLAY]

(2) Enter your number into the display.

(3) Press [=] [K( )] [0] [0] (store into K_{00})

or [=] [K( )] [0] [1] (store into K_{01})

or [=] [K( )] [0] [2] (store into K_{02})

•
•
•

or [=] [K( )] [2] [5] (store into K_{25})

TO READ OUT A NUMBER

Press [K( )] [0] [0] (readout from K_{00})

or [K( )] [0] [1] (readout from K_{01})

or [K( )] [0] [2] (readout from K_{02})

•
•
•

or [K( )] [2] [5] (readout from K_{25})

When you store a number into a box, the previous content of the box is <u>erased</u>.

When you read out a number from a box the number is <u>not</u> erased . . . it is still in the memory location.

1.01^n REVISITED

On Page 29, we defined a function

$$f(n) = 1.01^n$$

Here it is again.

CLEAR

DEFINE ● f(X) ●

= | K( ) | 0 | 0 | CLEAR DISPLAY | 1 | . | 0 | 1 | $|x|^y$ | K( ) | 0 | 0 | =

DEFINE ● f(X) ●

To use the function.

(1) Press CLEAR or CLEAR DISPLAY

(2) Enter the value of n.

(3) Press f(X)

(4) Read the value of 1.01^n in the display.

For example, suppose n = 25.

CLEAR | 2 | 5 | f(X)   1.282431995

Again for n = 99.

CLEAR DISPLAY | 9 | 9 | f(X)   2.678033494

When we enter a value of n and press [f(x)], the SCIENTIST

stores n into K_{00} | computes $1.01^{K_{00}}$

[=] [K( )] [0] [0] | [CLEAR DISPLAY] [1] [.] [0] [1] [|x|^y] [K( )] [0] [0] [=]

and-displays the result

Since the value of n is stored in K_{00},

$$1.01^{K_{00}} = 1.01^{n}$$

Let's expand the problem.

If we start with an initial population P_0 and the population increases r% per year for n years the final population is

$$\boxed{P_n = P_0(1 + \frac{r}{100})^n}$$

Try it for $P_0 = 1000$ people, $r = 2\%$ and $n = 10$ years.

FIRST, enter P_0 into K_{00}, r into K_{01} and n into K_{02}.

[CLEAR DISPLAY] [1] [0] [0] [0] [=] [K( )] [0] [0] — $1000 = K_{00}$

[CLEAR DISPLAY] [2] [=] [K( )] [0] [1] — $2 = K_{01}$

[CLEAR DISPLAY] [1] [0] [=] [K( )] [0] [2] — $10 = K_{02}$

Then, define the function, $P_n = P_0(1 + \frac{r}{100})^n$.

PRESS THE KEY(S)	AND CHECK THE DISPLAY
CLEAR	
DEFINE ●f(X)●	
K( ) 0 0	1000
×	
(1 + K( ) 0 1 ÷ 1 0 0)	1.02
$\lvert x \rvert^y$	
K( ) 0 2	10
=	1218.99442
DEFINE ●f(X)●	

$$\left\langle P_0(1 + \frac{r}{100})^n = 1000(1 + \frac{2}{100})^n \right\rangle$$

Use it. Press CLEAR f(X) <u>1218.99442</u>

In other words, if we start with an initial population of 1000 people and the population increases 2% per year for 10 years, the final population is about 1219 people.

But what if $P_0 = 1000$, $r = 2\%$ and $n = 20$? Simply change n.

CLEAR DISPLAY	2	0	=	$K_{()}$	0	2

Then press | f(x) | The answer is 1485.947396

Call it 1486 people.

Remember . . . the value of P_0 is in K_{00}

the value of r is in K_{01}

the value of n is in K_{02}

Complete the table.

P_0	r	n	P_n
1000	2	30	______
1000	2	40	______
1000	3	10	______
1000	3	20	______
1000	3	30	______
3.5×10^9	1	10	______
3.5×10^9	1	20	______

Oh yes, you can use the same function to do compound interest calculations.

For example. $100 at 6% per year for 5 years.

Keys	
CLEAR DISPLAY, 1, 0, 0, =, K( ), 0, 0	Principal in K_{00}.
CLEAR DISPLAY, 6, =, K( ), 0, 1	Interest rate in K_{01}.
CLEAR DISPLAY, 5, =, K( ), 0, 2	Number of years in K_{02}.

Then press f(x) The answer is $133.8225578

Round it to the nearest penny $133.82

Now try $100 at 6% per year, compounded <u>monthly</u>, for 5 years. For this case, r = 6 ÷ 12 = .5% per month and n = 12 x 5 = 60 months.

Keys	
CLEAR DISPLAY, ., 5, =, K( ), 0, 1	Change r.
CLEAR DISPLAY, 6, 0, =, K( ), 0, 2	Change n.

Run it. Press f(x) Answer <u>134.8850152</u>

$100 at 6% per year compounded <u>yearly</u> for 5 years $133.82	COMPARE	$100 at 6% per year compounded <u>monthly</u> for 5 years. (60 months) $134.89

MONEY PROBLEMS

REPAYING A LOAN

Suppose we borrow \$1000 at 9% per year interest rate and repay it in 5 equal yearly payments. How much do we pay each year?

First, we need a formula.

$$\boxed{R = \frac{Pi(1 + i)^N}{(1 + i)^N - 1}}$$

In the formula,

P = principal (amount borrowed)

i = interest rate, per payment period, expressed in decimal form.

N = number of equal payments.

R = repayment amount, yearly.

For our case,

P = \$1000

i = .09 (9% in decimal form)

N = 5 (5 yearly payments)

Use the SCIENTIST to compute R (try it yourself).

$$R = \frac{1000 \times .09 \times (1 + .09)^5}{(1 + .09)^5 - 1} = \frac{1000 \times .09 \times 1.09^5}{(1.09^5 - 1)}$$

\$257.09 (rounded to the nearest penny)

If you didn't get 257.09, try this.

$$\boxed{\text{CLEAR}}\ \underbrace{\boxed{1}\boxed{0}\boxed{0}\boxed{0}}_{p}\ \boxed{\times}\ \underbrace{\boxed{\cdot}\boxed{0}\boxed{9}}_{i}\ \boxed{\times}\ \underbrace{\boxed{1}\boxed{\cdot}\boxed{0}\boxed{9}}_{(1+i)}\ \boxed{|x|^y}\ \underbrace{\boxed{5}}_{N}$$

$$\boxed{\div}$$

$$\boxed{(}\ \underbrace{\boxed{1}\boxed{\cdot}\boxed{0}\boxed{9}\boxed{|x|^y}\boxed{5}}_{(1+i)^N}\ \boxed{-}\ \boxed{1}\ \boxed{)}\ \boxed{=}\ \_\_\_\_\_\_\_\_\_\_$$

We shop around and obtain a lower interest rate of 8% per year (.08 in decimal form).

● P = 1000, i = .08, N = 5. What is R? __________

Next, try 7%.

● P = 1000, i = .07. N = 5. What is R? __________

We may decide to take an 8% interest rate, but repay the loan in 7 years instead of 5.

● P = 1000, i = .08, N = 7. What is R? __________

For each of the four cases discussed so far, how much do we pay in all ? (Yearly payment x number of payments.)

(1) i = .09, N = 5. Total = __________

(2) i = .08, N = 5. Total = __________

(3) i = .07, N = 5. Total = __________

(4) i = .08, N = 7. Total = __________

LET'S AUTOMATE

We'd like to explore the loan repayment problem more thoroughly, but we want the SCIENTIST to do more of the work. So we'll define a function so that we can enter the values of P, i and N, then press - f(x) - and the SCIENTIST will compute R. We'll start by assigning $K_{(\ )}$ registers for P, i, N and R.

GIVEN VALUES: $P = K_{00}$, $i = K_{01}$ and $N = K_{02}$
COMPUTED VALUE: $R = K_{10}$

Now our function is

$$R = \frac{Pi(1+i)^N}{(1+i)^N - 1} = \frac{K_{00}K_{01}(1+K_{01})^{K_{02}}}{(1+K_{01})^{K_{02}} - 1} = K_{10}$$

But we must rearrange it in more SCIENTIST - like notation.

$$K_{00} \times K_{01} \times (1 + K_{01})\ |x|^y\ K_{02} \div ((1 + K_{01})\ |x|^y\ K_{02} - 1) = K_{10}$$

$$P \times i \times (1 + i)^N \div ((1+i)^N - 1) = R$$

Do you understand our shorthand?

K_{00}	means	$\boxed{K_{(\ )}}\ \boxed{0}\ \boxed{0}$
x	means	$\boxed{\times}$
$\lvert x\rvert^y$	means	$\boxed{\lvert x\rvert^y}$

and so on.

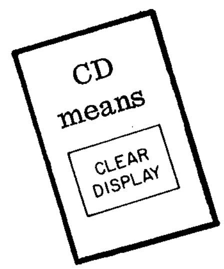
Well, we tried it, but it didn't work. Look at the function again.

$$K_{00} \text{ x } K_{01} \text{ x } (1 + K_{01})\ |x|^{y}\ K_{02} \div (\ (1 + K_{01})\ |x|^{y}\ K_{02} - 1) = K_{10}$$

Here is the trouble

The TEKTRONIX SCIENTIST does not permit parentheses to be <u>nested</u> in this manner. To be on the safe side, never use two or more left-hand parentheses [( ] consecutively. And don't use two or more right-hand parentheses [ )] consecutively. (But for exceptions, see THE TEKTRONIX SCIENTIST 909 OPERATING MANUAL).

Let's do it a different way. Like this:



(1) $K_{00} \text{ x } K_{01} \text{ x } (1 + K_{01})\ |x|^{y}\ K_{02} = K_{03}$

(2) CD $(1 + K_{01})\ |x|^{y}\ K_{02} - 1 = K_{04}$

(3) $K_{03} \div K_{04} = K_{10}$

Did you follow? Here's what we did.

(1) Computed the value of $K_{00}K_{01}(1 + K_{01})^{K_{02}}$ and put it into K_{03}.

(2) Computed the value of $(1 + K_{01})^{K_{02}} - 1$ and put it into K_{04}

(3) Number in K_{03} $\div$ number in K_{04} = final result. Put it into K_{10} and into the display.

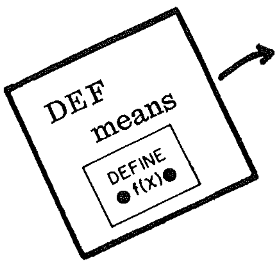
Now . . . enter the given data.

[CLEAR DISPLAY] [1] [0] [0] [0] [=] [K( )] [0] [0]

[CLEAR DISPLAY] [.] [0] [9] [=] [K( )] [0] [1]

[CLEAR DISPLAY] [5] [=] [K( )] [0] [2]

Then define the function.



CLEAR DEF

$K_{00} \times K_{01} \times (1 + K_{01})\ |x|^{y}\ K_{02} = K_{03}$

CD $(1 + K_{01})\ |x|^{y}\ K_{02} - 1 = K_{04}$

$K_{03} \div K_{04} = K_{10}$

DEF

Try it. Press [f(x)] <u>257.092457</u>

Then do these. (Don't forget the CD's.)

- $1000 = K_{00}$, $.08 = K_{01}$, $5 = K_{02}$ [f(x)] ____________
- $1000 = K_{00}$, $.07 = K_{01}$, $5 = K_{02}$ [f(x)] ____________
- $25000 = K_{00}$, $.075 = K_{01}$, $30 = K_{02}$ [f(x)] ____________

Cut loose! Enter <u>your</u> values and press [f(x)]

- ______ $= K_{00}$, ______ $= K_{01}$, ______ $= K_{02}$ [f(x)] ____________

SAME FUNCTION . . . ANOTHER WAY

Lest you forget . . . there's always another way.

CD 1000 = K_{00} CD .09 = K_{01} CD 5 = K_{02}
CLEAR DEF
$(1 + K_{01})\ \lvert x \rvert^{y}\ K_{02} = K_{03}$
$K_{00} \times K_{01} \times K_{03} \div (K_{03} - 1) = K_{10}$
DEF

Press $\boxed{f(x)}$ <u>257.092457</u>

Did you follow? When you press - f(x) -, the SCIENTIST evaluates $(1 + i)^N$ saves it, then uses it twice in computing the final result. This method requires fewer steps than the one on the preceding page.

- But suppose we want to pay in equal <u>monthly</u> installments, or quarterly installments, or semi-annual installments.

- And, we prefer entering the value of i as 9 instead of .09, or as 7.5 instead of .075 and so on.

First, the problem of entering i.   Easy!

CD 1000 = K_{00}	CD 9 = K_{01}	CD 5 = K_{02}

CLEAR DEF

$K_{01} \div 100 = K_{04}$

CD $(1 + K_{04})$ $|x|^y$ $K_{02} = K_{03}$

$K_{00} \times K_{04} \times K_{03} \div (K_{03} - 1) = K_{10}$

DEF

Compare the above with its counterpart on Page 46 and underline <u>all</u> differences.

Try the new program for P = \$5000, i = 6%, N = 10 years.

CD	5000 = K_{00}
CD	6 = K_{01}
CD	10 = K_{02}

f(x)   <u>679.3397911</u>

We pay \$679.34 per year.

P = \$3500, i = 6 3/4%, N = 12 years? ____________________

But we want to borrow \$10000 for 20 years at 9% per year and repay it with equal monthly payments. For this example,

$P = \$10000$

$i \approx 9\%$ per year $= .75\%$ per month

$N = 20$ years $= 20 \times 12 = 240$ months.

CD	$10000 = K_{00}$	f(x)	89.97259562
CD	$.75 = K_{01}$		
CD	$240 = K_{02}$		

My monthly payment is \$89.97.

Again $P = \$10000$

$i = 8.3\%$ per year

$N = 7$ years

Watch carefully.

CD	$10000 = K_{00}$	f(x)	157.36
CD	$8.3 \div 12 = K_{01}$		
CD	$7 \times 12 = K_{02}$		

Your turn. $P = \$3600$, $i = 7.59\%$ per year, $N = 23$ years.

Your monthly payment? ________________

Does all this give you an idea for an original program designed by you?

NUMBER PATTERNS

1, 2, 3, . . .

Teach the SCIENTIST to count. Follow our directions.

First, put zero into K_{00}.

[CLEAR DISPLAY] [=] [K( )] [0] [0]

THEN PRESS THE KEYS BELOW AND WATCH THE DISPLAY.

Keys	Display
[K( )] [0] [0] [+] [1] [=] [K( )] [0] [0]	1
[K( )] [0] [0] [+] [1] [=] [K( )] [0] [0]	2
[K( )] [0] [0] [+] [1] [=] [K( )] [0] [0]	3

and so on for as long as you wish!

Each time you press the keys

[K( )] [0] [0] [+] [1] [=] [K( )] [0] [0]

you cause the SCIENTIST to increase the content of K_{00} by 1. Like this:

[K( )] [0] [0]	[+] [1]	[=] [K( )] [0] [0]
Get the number from K_{00}	add 1 to it	put the result into the display and into K_{00}.

Let's make it more automatic.

[CLEAR] [DEFINE ●f(x)●] [K( )] [0] [0] [+] [1] [=] [K( )] [0] [0] [DEFINE ●f(x)●]

Put zero (0) into K_{00}.

[CLEAR DISPLAY] [=] [K( )] [0] [0]

Then watch the display and press [f(x)] several times.

[f(x)]	1
[f(x)]	2
[f(x)]	3

and so on!

Again . . . put zero into K_{00}, then watch the display and press - f(x) - several times. All together now: 1, 2, 3, 4, 5, . . .

EXPERIMENT!

Instead of putting zero into K_{00}, try some other number . . . a number chosen by you.

Instead of 1, 2, 3, . . .

let's try for 1, 4, 9, . . . $(1^2, 2^2, 3^2, \ldots)$

[CLEAR] [DEFINE ● f(x) ●] [K( )] [0] [0] [+] [1] [=] [K( )] [0] [0] [x^2] [DEFINE ● f(x) ●]

Put zero (0) into K_{00}.

[CLEAR DISPLAY] [=] [K( )] [0] [0]

Then press [f(x)] several times . . . watch the display.

[f(x)]	1
[f(x)]	4
[f(x)]	9

and so on.

Each time you press the - f(x) - key, the SCIENTIST does the following.

[K( )] [0] [0]	[+] [1]	[=] [K( )] [0] [0]	[x^2]
Get the number from K_{00}	add one to it	put the result into the display and into K_{00}	*

Then compute the square of the number in the display and put the answer in the display.

GUESS . . . THEN TRY IT!

For each problem, guess at the first three results *** write them down *** then run the problem on the SCIENTIST. If your guesses are correct, congratulate yourself. Otherwise . . .

- CD = K_{00}

  CLEAR DEF K_{00} + 2 = K_{00} DEF

  f(x) __________ f(x) __________ f(x) __________

- CD 3 = K_{25}

  CLEAR DEF K_{25} - 1 = K_{25} DEF

  f(x) __________ f(x) __________ f(x) __________

- CD 1 = K_{15}

  CLEAR DEF K_{15} x 2 = K_{15} DEF

  f(x) __________ f(x) __________ f(x) __________

Then complete this one - write in the function between DEF and DEF.
Here

- CD = K_{00}

  CLEAR DEF ____________________

  f(x) 1 f(x) .5 f(x) .3333333333

THE OLD CHESS BOARD PROBLEM

We found the following on Page 173 of Mathematics and the Imagination by Edward Kasner and James Newman. (Published by Simon and Schuster, New York.) *

> According to an old tale, the Grand Vizier Sissa Ben Dahir was granted a boon for having invented chess for the Indian King, Sirham. Since this game is played on a board with 64 squares, Sissa addressed the king: "Majesty, give me a grain of wheat to place on the first square, and two grains of wheat to place on the second square, and four grains of wheat to place on the third, and eight grains of wheat to place on the fourth, and so, Oh King, let me cover each of the 64 squares of the board."

The question, of course, is: How many grains of wheat are required to cover the entire board. Let's list a few squares and the number of grains on each square.

SQUARE	GRAINS
1	1
2	2
3	4
4	8
5	16
6	32
7	64
.	.
.	.
.	.

and so on.

> Each square has twice as many grains as the preceding square.

or

> Square k has twice as many grains as square k - 1.

How many grains on square #8? ______________________

How many grains on square #10? ______________________

Let the SCIENTIST do it. But first, take pencil in hand and follow the instructions below. The instructions refer to the following two boxes, K_{00} and K_{01}.

K_{00} K_{01}

DIRECTIONS - Use pencil because you will have to erase.

- Do this once.

  > Put 1 into K_{00} and also into K_{01}.

- Do the following several times. Each time, after carrying out the instructions, record the numbers in K_{00} and K_{01} in the table at the bottom of the page.

  > Increase the number in K_{00} by 1.

  > Double the number in K_{01}.

	K_{00}	K_{01}
FIRST TIME		
SECOND TIME		
THIRD TIME		
FOURTH TIME		
FIFTH TIME		
SIXTH TIME		
SEVENTH TIME		

Now it's the SCIENTIST's turn.

CD $1 = K_{00} = K_{01}$
CLEAR DEF $K_{00} + 1 = K_{00}$ $K_{01} \times 2 = K_{01}$ DEF

DISPLAY =	2	2 grains on square #2
Press K_{00}	2	
Press f(x)	4	4 grains on square #3
Press K_{00}	3	
Press f(x)	8	8 grains on square #4
Press K_{00}	4	

- Press f(x) several times . . . as many as <u>you</u> want.

DISPLAY ____________ ________grains
on
Press K_{00} ____________ square # _____

- Start again with CD $1 = K_{00} = K_{01}$

To find the number of grains on square N, press f(x), N - 1 times. For example, for N = 10 press f(x) nine times.

512

f(x) f(x) f(x) f(x) f(x) f(x) f(x) f(x) f(x)

Verify the number of the square . . . press K_{00} 10

We have 512 grains on square number 10.

? How many grains on square #23 ? ____________________

Another way.

CD 1 = K_{00} = K_{01}
CLEAR DEF K_{01} x 2 = K_{01} K_{00} + 1 = K_{00} DEF

Now when you press f(x), the number that pops into the display is the number of the square. To get the number of grains on that square, press K_{01}.

How many grains on square #16 ? ____________________

square #20 ? ____________ square #30 ? ____________

square #40 ? ____________ square #64 ? ____________

How many grains (total) on the first N squares? <u>You</u> choose N and use this program to help you get the answer.

CD 1 = K_{00} = K_{01} CD = K_{02}
CLEAR DEF
K_{01} x 2 = $K_{01}$$K_{01}$ + K_{02} = $K_{02}$$K_{00}$ + 1 = K_{00}
DEF

One more thing. Perhaps you have noticed that there are 2^{N-1} grains on square N. This fact may cause you to scrub <u>all</u> of our programs and write your own!

THE BEGINNING OF THE END OF THE BEGINNING

ANOTHER LOOK AT DOUBLING TIME

Remember this problem?

> If something (population, money or whatever) increases at the rate of r% per year, how long will it take to double? (See Page 27.)

The problem leads to the following equation.

$$(1 + r/100)^n = 2$$

where n is the doubling time in years.

We have already solved this problem two ways. If you have forgotten, review Pages 27 through 30. Let's do it still another way. First, some definitions and memory assignments.

$$f(n) = (1 + r/100)^n$$

We assume

$r = K_{00}$

$n = K_{01}$

$\Delta n = K_{02}$ (Δn is the amount by which n changes each time we press f(x)).

$f(n) = K_{03}$

Now, define a function for the SCIENTIST.

CLEAR DEF	
$K_{01} + K_{02} = K_{01}$	← increase n by Δn.
CD $(1 + K_{00} \div 100)$ $\|x\|^y$ $K_{01} = K_{03}$	← compute f(n).
DEF	

Let's try it for r = 2%, initial value of n = 0 and $\Delta n = 10$.

CD 2 = K_{00}	CD = K_{01}	CD 10 = K_{02}
r	n	Δn

PRESS f(x) and read DISPLAY.

f(x)	1.21899442	Less than 2.	Try again.
f(x)	1.485947396	Less than 2.	Try again.
f(x)	1.811361584	Less than 2.	Try again.
f(x)	2.208039663	More than 2.	Let's discuss it.

We claim that, inside the SCIENTIST the value of n is now 40. Let's find out.

Press K_{01} 40 Right on!

Now think about how n got to be 40. It started at 0 and was increased by 10 each time we pressed f(x). Therefore, the value of n that we are looking for is between 30 and 40. Let's reset n to 30 and change Δn to 1.

CD 30 = K_{01}	CD 1 = K_{02}

f(x)	1.847588815	Less than 2.	Try again.
f(x)	1.884540592	Less than 2.	Try again.
f(x)	1.922231404	Less than 2.	Try again.
f(x)	1.960676032	Less than 2.	Try again.
f(x)	1.999889552	Less than 2.	Try again.
f(x)	2.039887343	More than 2.	

Press K_{01} 36. The value of n is now 36.

It looks as if the desired value of n is between 35 and 36 but closer to 35. We will accept 35 as a close enough answer.

Your turn. For each value of r, find the doubling time to the nearest 10th of a year.

$r = 3$ Doubling time = ____________________ years.

$r = 2.3$ Doubling time = ____________________ years.

We designed a SCIENTIST method to help answer the question:
For a given value of r, what value of n satisfies the equation

$$(1 + r/100)^n = 2?$$

Your job, should you choose to accept it, is to redesign our method to help answer the converse question: For a given value of n, what value of r satisfies the equation.

$$(1 + r/100)^n = 2?$$

Then use the SCIENTIST to complete the following. Compute r to the nearest .1%.

$n = 23$ years. $r =$ ____________________?

$n = 41$ years. $r =$ ____________________?

$n = 10$ years. $r =$ ____________________?

THE BETTER WAY STRIKES AGAIN

The doubling time equation is

$$(1 + r/100)^n = 2$$

If you made it through high school algebra, you must know something about <u>logarithms</u>. Let's apply a little logarithmic knowhow to the above equation.

log means logarithm, base 10

(1) $\quad \log[(1 + r/100)^n] = \log 2$

(2) $\quad n \log (1 + r/100) = \log 2$

(3) $\quad n = \dfrac{\log 2}{\log (1 + r/100)}$

Number (3) is what we are after. It gives us a formula for n in terms of r.

Let the SCIENTIST do it for r = 2%. First, find the $\boxed{\log x}$ key. We will abbreviate it LOG.

CLEAR 2 LOG ÷ (1 + $\underbrace{2}_{r}$ ÷ 100)LOG = $\underbrace{35.00278879}_{n}$

As we expected, $n \cong 35$ years.

Again, for r = 3%

CLEAR 2 LOG ÷ (1 + 3 ÷ 100) LOG = 23.44977225

To the nearest 10th of a year, n = 23.4 years.

log x	compute the logarithm, base 10, of the number in the display and put the result into the display.

Let's trace what happens as we press the keys for r = 3.

KEY(S)	DISPLAY	REMARKS
CLEAR	0	
2	2	
log x	.3010299957	log 2
÷	0	
(	0	
1	1	
+	0	
3	3	(1 + r/100)
÷	0	
1 0 0	100	
)	1.03	
log x	.0128372247	log (1 + r/100)
=	23.44977225	n

Store the formula for n as a SCIENTIST function.

CLEAR 3 DEF

= K_{00} CD 2 LOG ÷ (1 + K_{00} ÷ 100) LOG = K_{01}

DEF

To use the above function,

- Press CD (CLEAR DISPLAY)
- Enter r
- Press f(x) and read answer.

CD 2 f(x) 35.00278879 years

CD 4 f(x) 17.67298769 years

Your turn

(1) r = 5, n = ______ (2) r = 3.7, n = ______

In defining the function, why did we begin with

CLEAR 3 DEF instead of simply CLEAR DEF?

Try it using CLEAR DEF. What happens? Why?

We used the log, base 10 key. There is also a key to compute log, base e. It looks like this

$\boxed{\text{ln x}}$

Use it to compute n for a given value of r instead of using

$\boxed{\text{log x}}$

EXTRA FOR EXPERTS

There is also a better way to compute r, given n. Again, we start with the equation

$$(1 + r/100)^n = 2.$$

Then apply a little mathematical knowhow. This time, we use natural logarithms, (logarithms, base e).

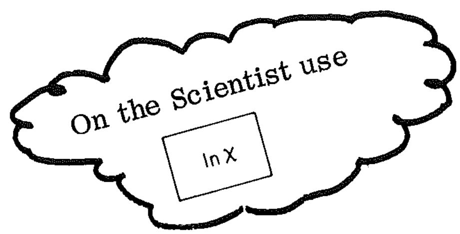
(1) $\ln[(1 + r/100)^n] = \ln 2$

(2) $n \ln (1 + r/100) = \ln 2$

(3) $\ln(1 + r/100) = \ln 2 / n$

(4) $1 + r/100 = e^{\ln 2/n}$

(5) $r = 100(e^{\ln 2/n} - 1)$



We know that if n = 35, then r is very close to 2. Here are three different ways to compute r.

CLEAR	(2 LN	35) e^x - 1	= x 100 =	2.000160941
CLEAR	2LN	35 = e^x - 1	= x 100 =	2.000160941
CLEAR	(2LN	35 = e^x - 1)	x 100 =	2.000160941

Use any of the above methods. Define it as a TEKTRONIX SCIENTIST function in a manner similar to our function for n as a function of r on Page 62. Then use the function to compute r for each value of n below.

n = 25, r = ________ n = 10, r = ________

JANUS

Janus is a god in Roman Mythology. He has two faces, one facing backward and one facing forward. Let's look backward, look forward.

- We have used some, but not all of the keys on the SCIENTIST's keyboard.

- We have introduced a number of problem-solving techniques. But there are many more.

- We have used only the minimum SCIENTIST. We have not discussed the many peripherals which can be attached to it.

  Such as PRINTER, PROGRAMMER, INSTRUCTOR, CARD READER, f(x) REPEATER, XY PLOTTER, and so on.

- We haven't mentioned the growing PROGRAM LIBRARY. The program library is a collection of programs to solve problems in many fields of application.

This is the end of the beginning. From here you are on your own. Look ahead . . . begin by perusing the following.

- THE TEKTRONIX SCIENTIST 909 OPERATING MANUAL

- PROGRAM LIBRARY for the TEKTRONIX SCIENTIST 909 COMPUTING CALCULATOR.