LECTURE NOTES

TRANSIENTS

I. Introduction

A. Pulse Terminology

1. Period: Time for one complete cycle of values.

2. Repetition Rate (Freq.): $\frac{1}{\text{Period}}$ $\frac{1}{\text{Time}}$

3. Pulse Duration: Interval between the first and last instants the pulse reaches a specified percent of PK.

4. Duty Cycle: $\frac{\text{Duration}}{\text{Period}}$ Also duration X rep rate usually expressed in percentage.

5. Amplitude: Peak to peak; usual method

a. Average DC; as read by meters, shows ability to charge a battery or filter capacitor.

b. Effective; RMS shows AVERAGE POWER content, not just "E".

B. Transient Definition

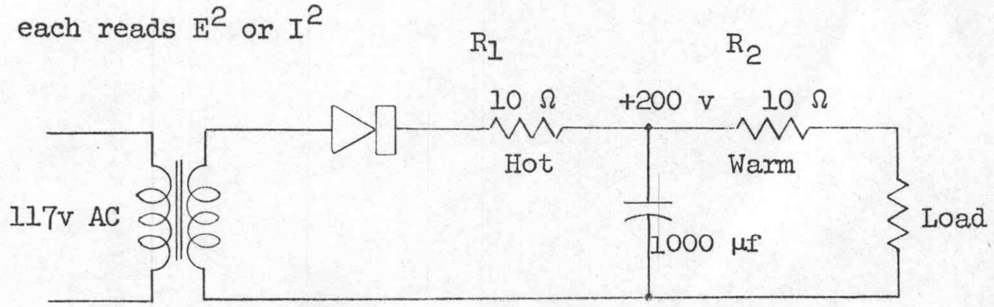
1. A temporary or changing condition - "Webster".

2. A rapidly changing amplitude of E or I in a circuit.

C. Effective voltage of pulse

1. Read by Thermo-Coupled Meter or Moving Iron Vane Meter; each reads E^2 or I^2

2.



a. The average current thru both resistors are the same. Because of its pulse nature, the effective current is greater in R_1.

II. Capacitors

A. Capacitance

1. A capacitor has a capacitance of one farad when one coulomb of electricity charges it to one volt.

2. $C = \frac{Q}{E}$ or $Q = CE$

3. $C = 0.2246 \frac{KA}{S}$ where K is the dielectric constant, A is the area of the plates in square inches, and S is the distance between the plates in inches.

B. Capacitors in parallel

1. Add directly $(C_t = C_1 + C_2 \text{ ---})$

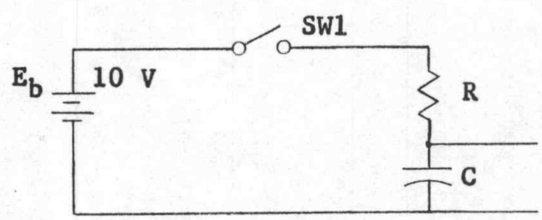
C. Capacitors in Series

1. Add by reciprocals $\left(C_t = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} \text{ ---}}\right)$

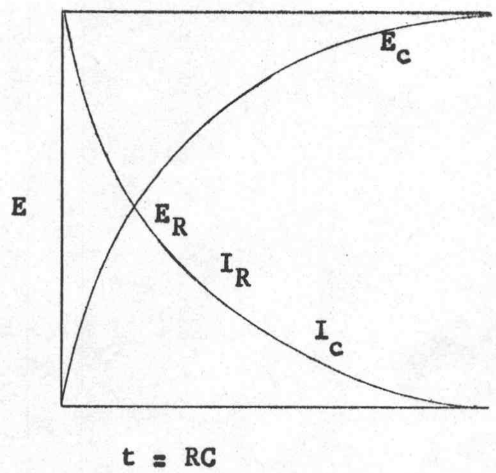
III. RC Time

A. Capacitor Charging

1. Capacitor charging circuit



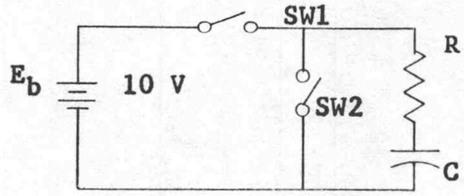
2. Charging curve



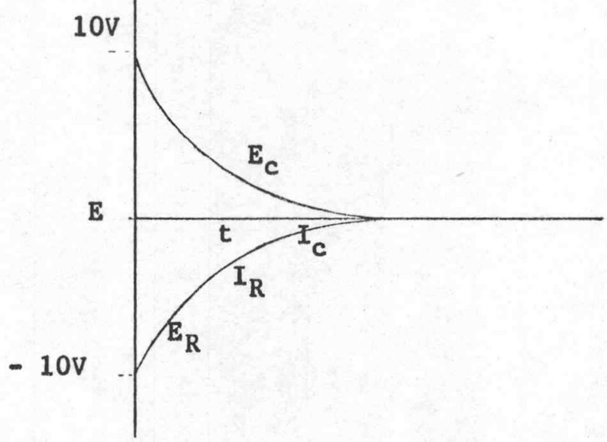
3. As SW_1 is closed (at zero time), C appears as a short.
   a. The entire 10 volts appears across R, and $I = \frac{10V}{R}$
   b. The current into C is limited only by R.
4. As electrons flow out of the top plate of the capacitor through R, a voltage drop appears across the capacitor.
   a. A voltage division is formed, composed of E_R and E_C (the sum must always equal E_B)
   b. As E_C increases, E_R decreases and I decreases at the same rate.
5. With the current into the capacitor decreasing, the "rate of charge" decreases at an exponential rate.

B. Capacitor Discharge

1. Capacitor discharge circuit



2. Discharge curve



3. Consider the capacitor completely charged, then open SW 1 and close SW 2.

   a. At zero time, $E_C = 10$ v.

   b. $E_c + E_R$ must equal 0 v.

   c. Therefore $E_R = -10$ v

   d. C appears as a 10 v battery in series with R -- the current limited only by R.

4. The current (beginning as a maximum negative current) will decline exponentially.

   a. E_c and E_R will decay at the same exponential rate.

   b. At any instant $E_c + E_R$ must equal 0.

C. Rate of Capacitor Charge and Discharge

1. If there is voltage across a capacitor, it has two things stored.

   a. Energy, in joules, or watt seconds ($1/2\ E^2\ C$)

   b. Charge, in coulombs, or ampere seconds (EC)

      (1) The capacitor tries to keep its voltage the same.

2. If voltage is applied to a capacitor, electrons will be removed from one plate while electrons are added to the other.

   a. The apparent current flow is called "displacement current".

   b. The effect is current **flowing** through the capacitor, and may be so considered although not actually true.

   c. $i = C\ \frac{de}{dt}$

   d. $\frac{de}{dt} = \frac{\text{volts}}{\text{sec}}$

3. Current Waveform amplitude will be proportional to the slope or rate of charge of voltage.

   a. Example:

      $i = C\ \frac{de}{dt}$

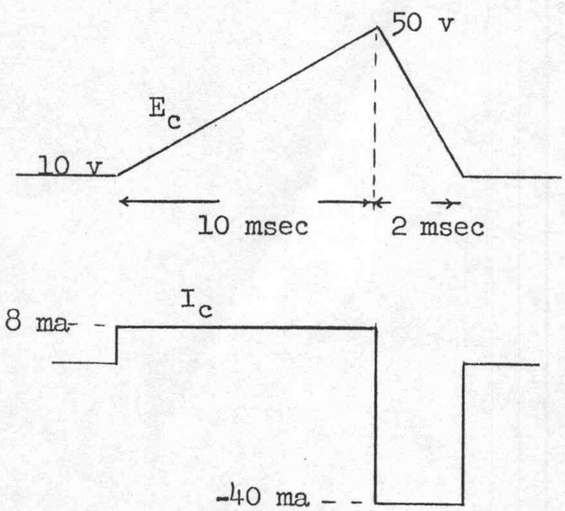
      let $C = 2\ \mu f$

      $i = 2\ \mu f \times \frac{40\ v}{10\ msec}$

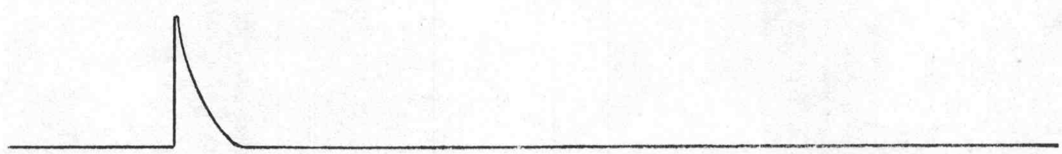
      $i = 8\ ma$

      $i = 2\ \mu f \times -\frac{40\ v}{2\ msec}$

      $i = -\ 40\ ma$



4. For time constants less than 10 times the pulse duration, the amount of tilt (differentiation) should be calculated.
   a. For TC of 10 times or more, a good rule of thumb will allow one to consider the reciprocal of the time constant ratio as the tilt ratio.
   b. Example: a pulse duration of 10 msec, when applied to a network with a TC of 100 msec, will suffer a 10% tilt.
5. Note that a change (step) in D.C. level will result in a pulse (the step will have been differentiated) followed by a return to the level of the resistor return.



6. A long time constant is considered as one whose duration exceeds 10 times the pulse duration.
7. A short time constant has a duration less than one tenth that of the pulse duration.

8. Rate of discharge of a capacitor for the first 10% of discharge.

a. Rate of charge $= \frac{de}{dt} = \frac{I}{C}$

since $I = \frac{E}{R}$, $\frac{de}{dt} = \frac{E}{RC}$

where E is original voltage

and I is I original provided time is short.

transposing $\frac{dt}{RC} = \frac{de}{E}$

b. For the first 10% of discharge

% of E ≅ % of RC

c. Example--in .017 TC the capacitor will have discharged .017 of the total amount.

D. Calculation of RC Time

1. TC = RC (the product of R in ohms and C in farads)

   a. At 1 TC, E_c = 63% of EB

      (1) This is 37% (actually 36.8%) left to charge

   b. At 2 TC, E_c = 86.5% or 13.5% left to go.

   c. At 5 TC, E_c = 99.5% and for practical purposes, can be considered fully charged.

2. Examples:

   a. If 1 TC = 4 sec it means that the capacitor has become 63% charged (or 36.8% left to charge) in 4 seconds.

   b. 2 TC = 8 sec. = .368 x .368 to go.

   c. 3 TC = 12 sec = $(.368)^3$ to go, etc.

   d. .57 TC = .57 x 4 = 2.28 sec = $(.368)^{.57}$ to go.

   e. $.368 = \frac{1}{\varepsilon} = \varepsilon^{-1}$, so in 1 TC it has ε^{-1} to go or in 7 TC it has ε^{-7} to go.

3. To find t for instantanious values of e_c and e_R

   a. $e_R = E\varepsilon^{-\frac{t}{Rc}}$, solve for t.

   Let E = 100 v, R = 500 Ω, and C = 20 μf and e_R = 40 v

$$40 = 100\ \varepsilon^{-\frac{t}{500 \times 20 \times 10^{-6}}}$$

$$40 = 100\ \varepsilon^{-\frac{t}{10-2}}$$

$$40 = 100\ \varepsilon^{-100}$$

$$.4 = \varepsilon^{-100t}$$

$$.4 = \varepsilon^{\frac{1}{100t}}$$

$$.4\ \varepsilon^{100t} = 1$$

$$\varepsilon^{100t} = 2.5$$

$$\log_{10} \varepsilon^{100t} = \log_{10} 2.5$$

$$100t \log_{10} \varepsilon = \log_{10} 2.5$$

$$100t \times .4343 = .398$$

$$t = \frac{.398}{43.43}$$

$$t = .00918 \text{ sec or } 9.18 \text{ msec}$$

b. $e_c = E(1 - \varepsilon^{-\frac{t}{R_c}})$

$$60 = 100\ (1 - \varepsilon^{-100t})$$

$$.6 = 1 - \varepsilon^{-100t}$$

$$.4 = \varepsilon^{-100t}$$

$$.4 = \frac{1}{\varepsilon^{100t}}$$

$$.4\ \varepsilon^{100t} = 1$$

$$\varepsilon^{100t} = 2.5$$

$$\log_{10} \varepsilon^{100t} = \log_{10} 2.5$$

$$100t \log_{10} \varepsilon = .398$$

$$100t \times .4343 = .398$$

$$t = \frac{.398}{43.43}$$

$$t = .00918 \text{ sec or } 9.18 \text{ msec.}$$

c. One can also solve for t, retaining RC.

$$e_R = \varepsilon^{-\frac{t}{RC}}$$

$$\frac{e_R}{E} = \varepsilon^{-\frac{t}{RC}}$$

$$\frac{E}{e_r} = \varepsilon^{\frac{t}{RC}}$$

$$\log_\varepsilon \frac{E}{e_r} = \frac{t}{RC}$$

$$t = RC \log_\varepsilon \frac{E}{e_r}$$

$$t = RC \log_\varepsilon \frac{100}{40}$$

$$t = RC \log_\varepsilon 2.5$$

$$t = .918 \text{ RC}$$

$$t = .918 \times 500 \times 20 \times 10^{-6}$$

$$t = .00918 \text{ sec or } 9.18 \text{ msec.}$$

4. Using same values as above, E_c has $\frac{40}{100}$ = 40% left to go.

   a. From the slide rule (LL00 or LL2 scale)

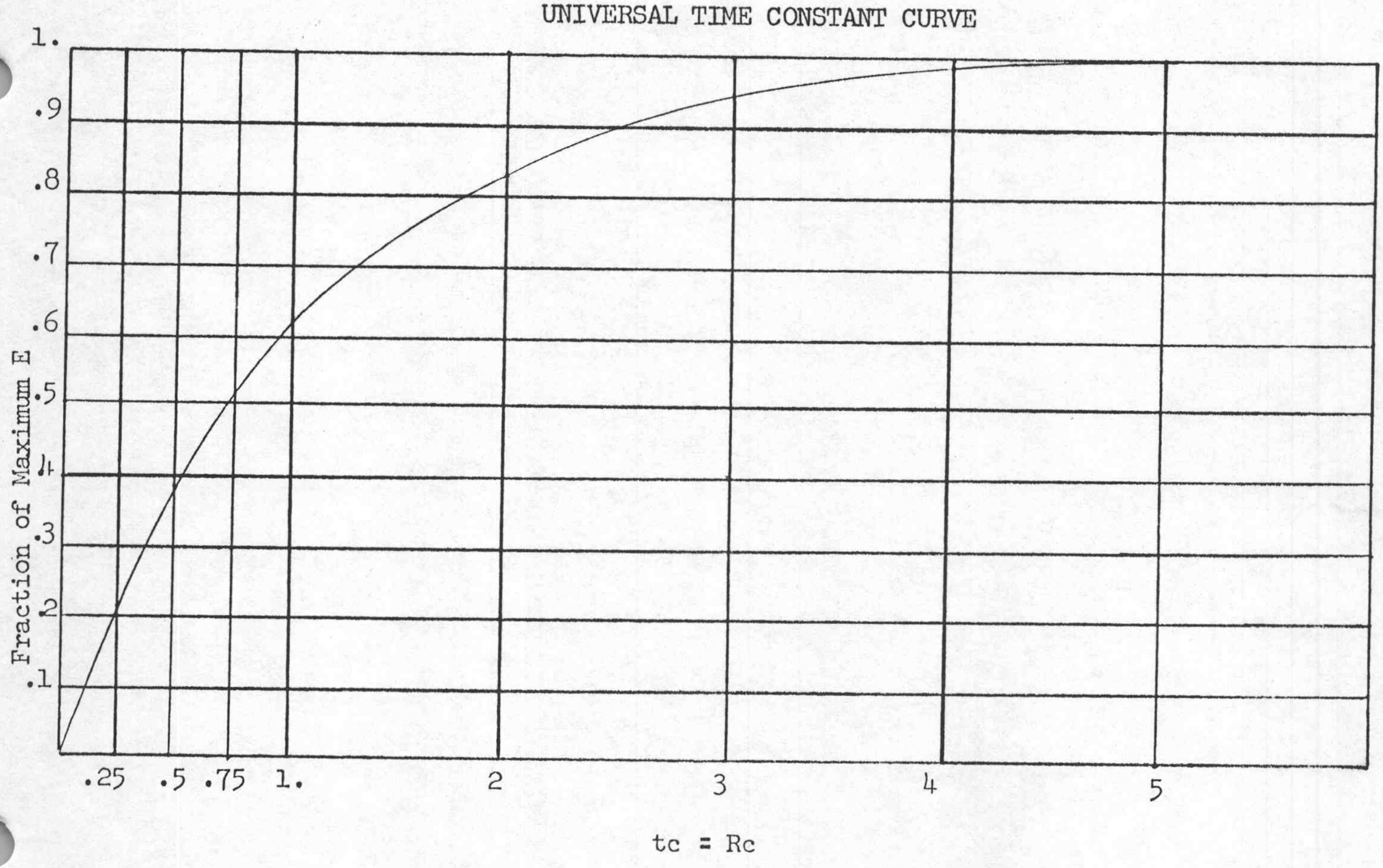
   40% = .916 TC

   b. TC = RC = 500 x 20 x 10^{-6} = 10^{-2} seconds or 10 msec.

   c. .916 x 10 = 9.16 msec.

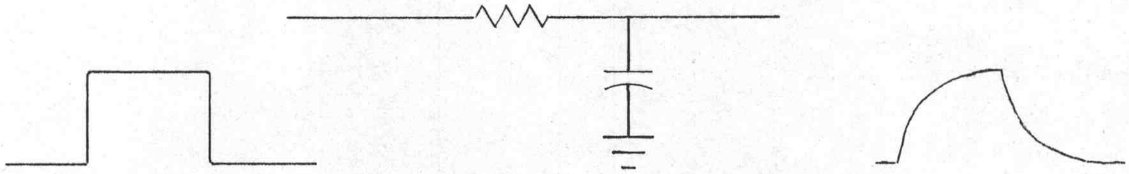
5. In lieu of a slide rule, a universal time constant curve can be used.

UNIVERSAL TIME CONSTANT CURVE



E. Integration

1. The simple integrating network is merely the basic RC circuit redrawn.



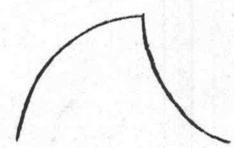
a. The square pulse can be considered the closing of SW 1 (pulse rise), and opening SW 1 and closing SW 2 (pulse fall).

b. The integrating waveform is the E_c charge and discharge curve.

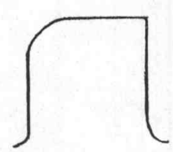
c. Trace electron flow.

d. The amount of integration depends upon the ratio of pulse duration to RC time.

(1) If the pulse duration is 10 msec and the RC time is 50 msec (5 times the pulse duration), the capacitor will just complete its charge at the end of the pulse.

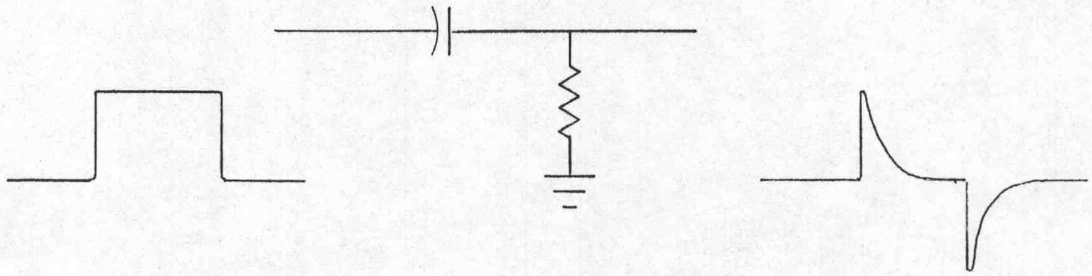


(2) If the pulse duration is 10 times the RC time the output waveform will appear slightly rounded.



F. Differentiation

1. The differentiation circuit is the same basic RC circuit, redrawn so the take-off is across the resistor.



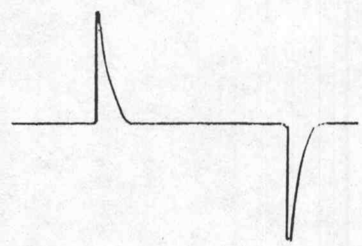
a. The output waveform is the C_R charge and discharge curve.

b. Trace electron flow

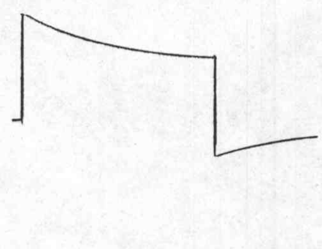
c. The amount of differentiation depends upon the ratio of pulse duration to RC time.

(1) If the pulse duration is 5 times the RC time, the capacitor will just become fully charged at the end of the pulse.

2. A shorter time constant would result in a shorter pulse.

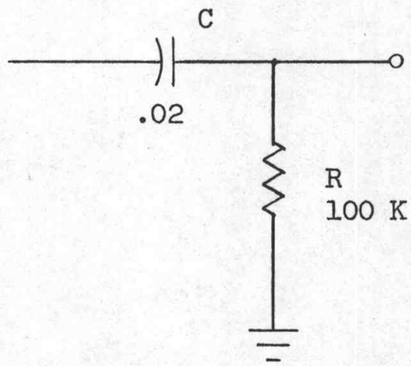


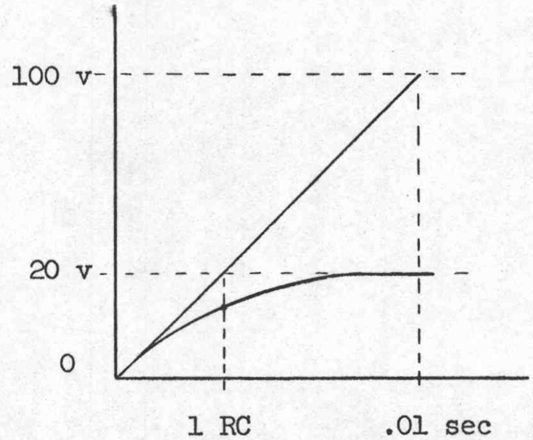
3. A longer time constant would not allow full differentiation--the resulting waveform is commonly referred to as "tilt".



G. Differentiating a Ramp.

1. Illustration:





2. Example -- a sawtooth waveform that reaches 100 v in .01 seconds is applied to the differentiating circuit.

   a. e_R will increase exponentially until at 1 RC it will be 63% of the applied voltage at that time (20 v).

   b. In 5 RC the voltage will level off at a value equal to the applied voltage at 1 RC (e_{max}).

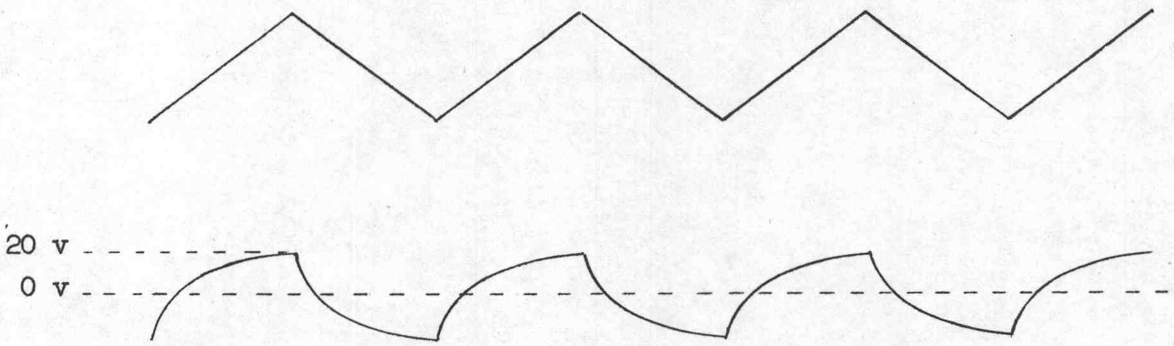
   c. $e_{max} = \frac{de}{dt} RC$

$$= \frac{100}{.01} \times 10^5 \times 2 \times 10^{-8}$$

$$= 20 \text{ v}$$

3. After many cycles the output will have an excursion about the DC average.

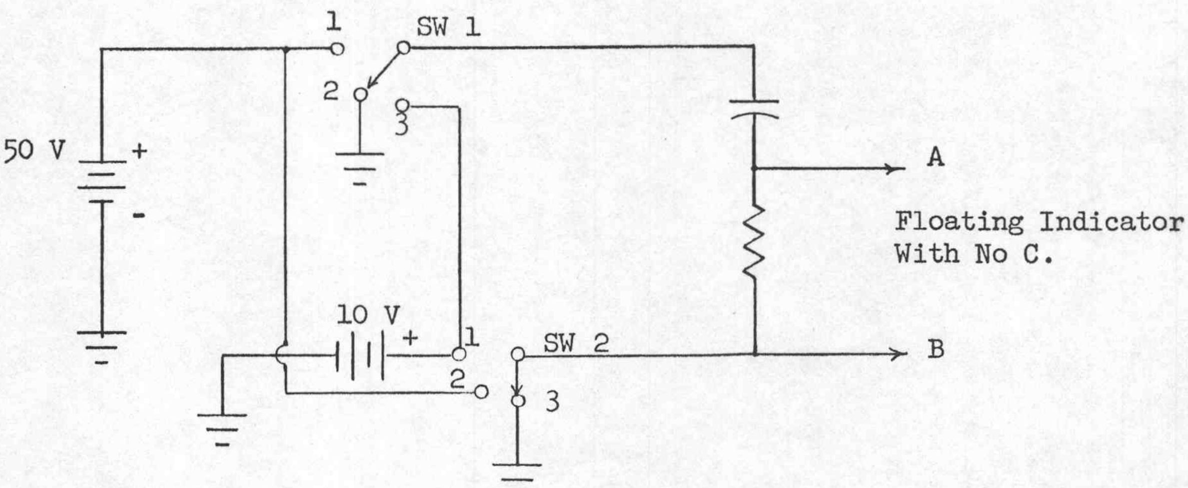
   a. Consider a sawtooth 100 v p-p and a TC of .02 sec.



4. A step with less than perfect rise time will react as a ramp when applied to a differentiating circuit with a short RC. The output waveform may have an amplitude something less than expected.

H. Integration and Differentiation Example.

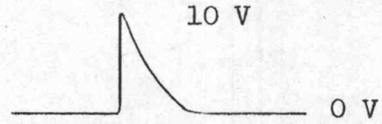
1. Capacitor charge and discharge can be illustrated by this sketch.



2. Waveforms

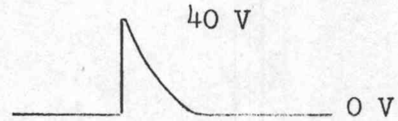
a. Initial condition--both switches to ground--

no charge on C.



b. SW 1 to position 3.

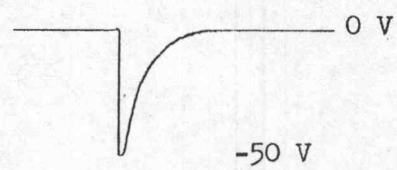
(1) output is integrated pulse.



c. SW 1 to position 1

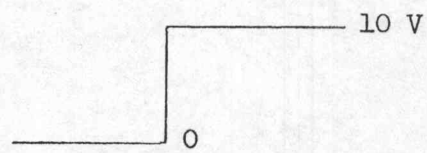
(1) C was charged to 10 v.

(2) Switching to 50 v adds additional 40 charge with 40 v step.



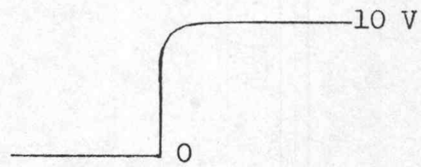
d. SW 1 to position 2.

(1) The entire charge on C is dumped, delivering 50 v negative step.

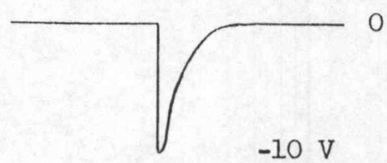


e. SW 2 to position 1.

(1) The bottom of the indicator has 10 v step.



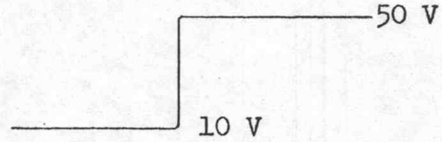
(2) The top of the indicator is integrated 10 v step.



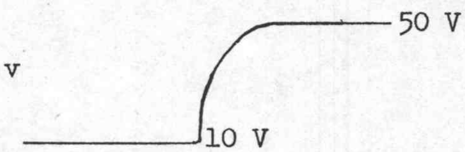
(3) The indicator would display the difference.

f. SW 2 to position 2.

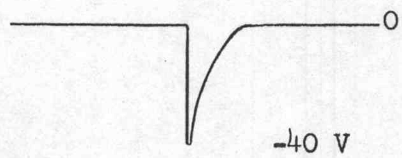
(1) The bottom of the indicator has 40 v step to 50 v.



(2) The top of the indicator has 40 v integrated step.

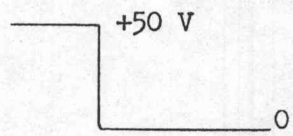


(3) The indicator displays the difference.

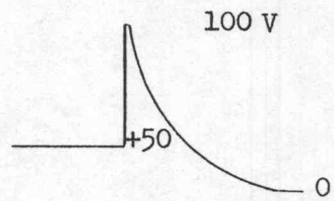


g. SW 2 to position 3, and SW 1 to position 1.

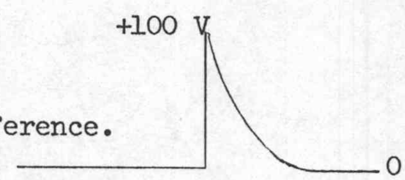
(1) The bottom of the indicator goes to ground



(2) The top of the indicator goes to 100 v then returns to ground exponentially.



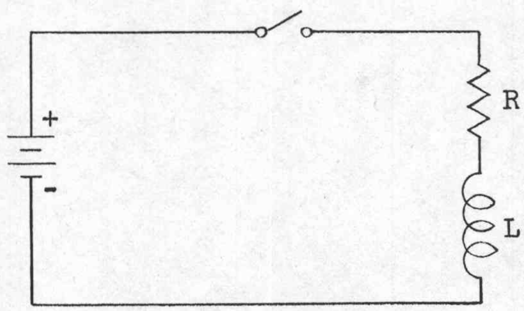
(3) The indicator displays the difference.

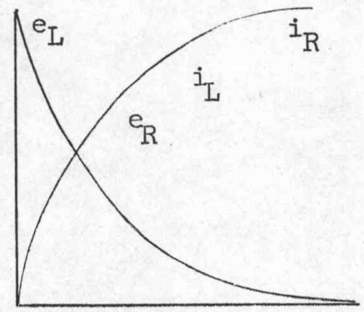


IV $\frac{L}{R}$ TIME

A. Inductor Charging.

I. Inductor charging circuit -- charging curve.





2. When sw_1 is closed (at zero time), counter emf prevents current flow.

   a. Initially no current flows thru R, so the entire voltage drop occurs across L.

   b. As current begins to flow into L, a voltage drop begins to appear across R.

   c. $e_L = L \frac{di}{dt}$, so e_L is a direct function of the slope or rate of change of current thru L.

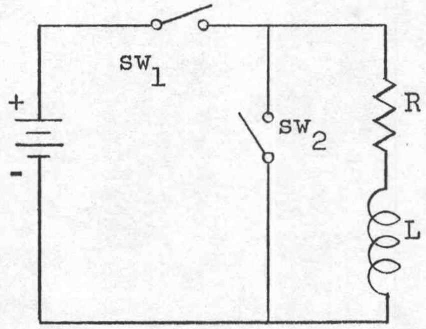
   d. At any instant, $e_L + e_R$ will equal the supply voltage.

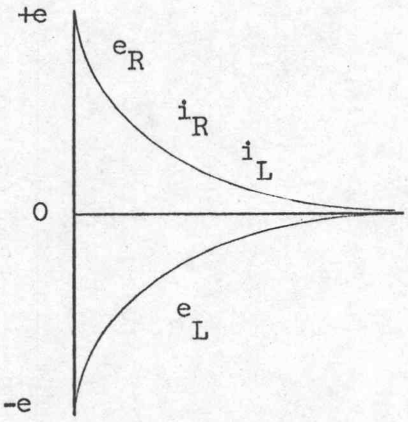
   e. After L is fully charged, any voltage drop across L can be considered as across the internal resistance of the inductance.

   f. $TC = \frac{L}{R}$.

B. Inductor Discharging

1. Inductor discharge ciucuit -- discharge curve.





2. When sw_1 is opened, and sw_2 is closed simultaneously the collapsing electromagnetic field causes current to flow.

   a. The amplitude of current is limited only by R.

   b. As the current flows thru R, a voltage drop occurs across R.

   c. The sum of the voltage drops across R and L must equal zero.

      (1) The voltage drop across L must, therefore, be a negative voltage, and the current a negative current.

   d. As the field collapses the current falls exponentially to zero.

      (1) All voltage drops are also returned to zero.

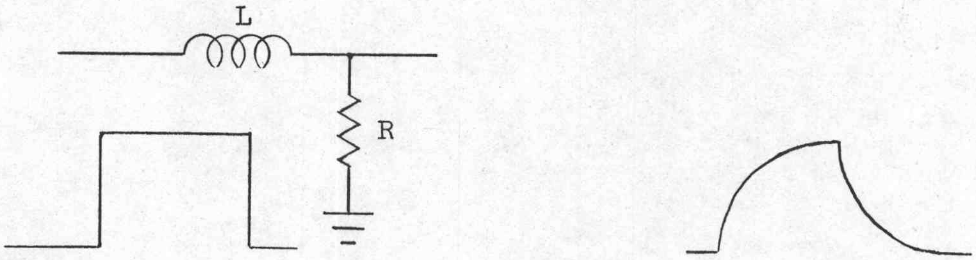
   e. As the slope of current is a negative slope, so must C_L be negative.

      (1) $C_L = L \frac{di}{dt}$, with di negative, C_L is negative.

C. Integration.

1. The simple $\frac{L}{R}$ integrating network has the same configuration as the RC differentiating network.

2. The sudden change or step cannot be instantly coupled thru L.

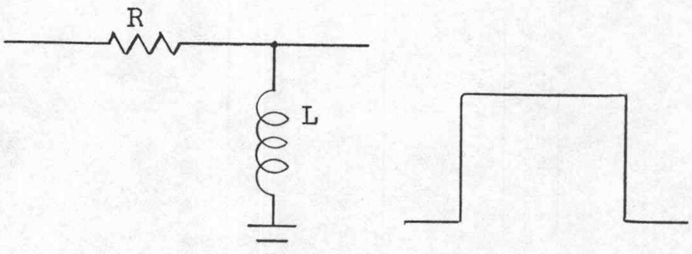


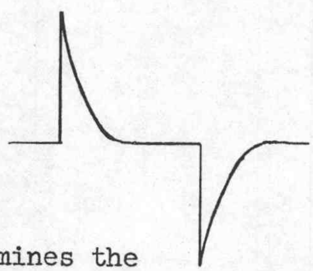
3. The $\frac{L}{R}$ time relative to pulse duration determines the degree of integration.

D. Differentiation.

1. The simple $\frac{L}{R}$ differentiating network has the same configuration as the RC integrating network.

2. The step couples thru R, and is then shunted to ground by L as soon as the high frequency component has passed.





3. The $\frac{L}{R}$ time relative to pulse duration determines the degree of differentiation.