NOISE MEASUREMENTS USING THE SPECTRUM ANALYZER

PART TWO: IMPULSE NOISE

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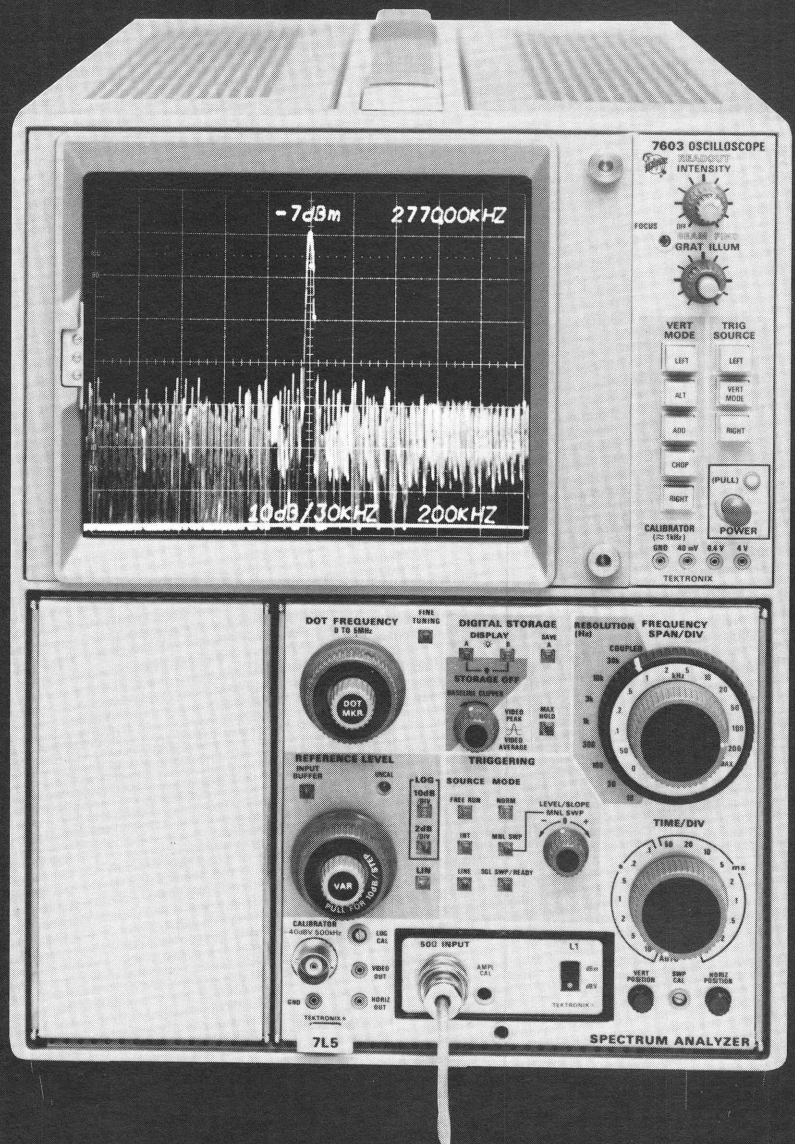
NOISE MEASUREMENTS USING THE SPECTRUM ANALYZER
Part Two: IMPULSE NOISE

By Morris Engelson

Impulses and pulses are always around us. Some are undesirable interference, such as stray bits in computers or ignition noise in automobiles. Other narrow pulses, such as in Loran-C or radar, perform a useful function. In either case, *the frequency distribution is an important parameter and must be measured.* This is easily done with spectrum analyzers. Most of this discussion applies to any spectrum analyzer, no matter what the make or model number. Specific illustrations and examples use the TEKTRONIX 7L5 Spectrum Analyzer. Although some of these data, such as on digital storage, is specific to the 7L5, the basic theory applies generally.

The paper is divided into six sections. The first defines impulse functions. Section two reviews methods for determining instrument impulse bandwidth. Section three considers the characteristics of practically occurring pulses. Section four presents procedures for operation within the linear dynamic range of the instrument. The fifth section considers the special effects resulting from digital storage. The paper concludes with a summary for quick reference.

I. The Impulse

The impulse or Dirac delta function, having zero time duration and infinite voltage amplitude, is a mathematical fiction. Such a function is not physically realizable. However, a circuit responds to a very narrow pulse as though it were such an impulse. Generally, a circuit of bandwidth "B" will respond to a pulse having time duration "t_0" as though t_0 were an impulse, when $t_0 B \leq 0.1$. *For our purposes, then, an impulse is simply a sufficiently narrow pulse.*

The Fourier transform of an impulse is equal to the impulse area, or strength. However, the frequency spectrum defined by the Fourier transform extends to negative as well as positive frequencies. Since negative frequencies do not exist, it follows that the impulse spectral intensity in volts/Hertz is numerically equal to twice the area of the impulse in volt-seconds.[1] Spectral intensity is a volts-per-frequency phenomenon. Hence, *to measure impulse spectral intensity one must know the impulse bandwidth of the measuring instrument.*

II. Impulse Bandwidth

Several ways to determine the impulse bandwidth of tuned amplifiers appear in the literature. Some of these are experimental, some theoretical, others are a combination of both. Many of the techniques are approximate, for use with specific circuitry only.

Several of these techniques that apply to spectrum analysis were tried out on the 3-kHz resolution bandwidth position of a TEKTRONIX 7L5 Spectrum Analyzer. Results are as follows:

The Integrated Voltage Bandwidth Method

$$\left(B = \frac{1}{G_0} \int_{-\infty}^{+\infty} \left| G(f) \right| d_f \right)$$

Theory—This experimental technique involves finding the average frequency width of the voltage response curve. This is easily done by *measuring the area under the voltage response curve and dividing by the peak response height.* This technique will yield a number that is the upper limit of the impulse bandwidth as the number of tuned stages increases.[2] The method should not be used for one- or two-stage amplifiers. For those circuits, it is quite inaccurate.

[1]*See Engelson & Telewski—"Spectrum Analyzer Theory and Application", Artech House 1974, for a discussion on the differences between the mathematical theory and practical applications.*

[2]*Geselowitz — "Response of Ideal Radio Noise Meter to Continuous Sine Wave, Recurrent Impulses, and Random Noise", IRE TRANS. on RFI, Vol. RF1-3 No. 1, May 1960*

Measurement—Figure 1 is the voltage response of the 7L5's 3kHz resolution bandwidth position. The area under this curve is 23.9 square divisions. Dividing by the height of 8 divisions and multiplying by the frequency span yields:

$$B = \frac{23.9 \ (\text{div})^2}{8 \ \text{div}} \times 1 \ \text{kHz/div} \simeq 3.0 \ \text{kHz}$$

The Impulse Response Method

$$B = \left(\frac{V_0}{\int_{-\infty}^{+\infty} \left| V(t) \right| \ dt} \right)$$

Theory—When a cascade of single tuned stages is excited by an impulse, the peak of the output pulse envelope is equal to the area under the output pulse envelope times the effective impulse bandwidth.[2] Hence, *the effective impulse bandwidth equals the ratio of the peak of the output pulse envelope in volts to the output pulse envelope area in volt-seconds.*
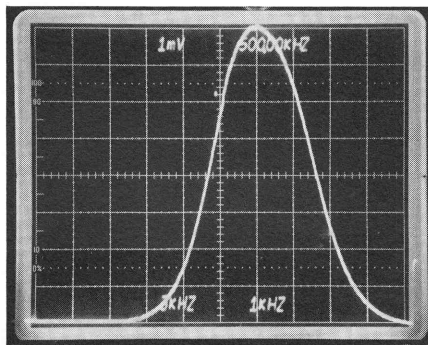
This experimental method is independent of the number of stages in cascade; however, errors may be introduced if stages are not single tuned.

Measurement—Figure 2 is the time response of the 7L5's 3 kHz resolution bandwidth position. The peak output response is 8 divisions, and the area under the curve is 25.5 square divisions with a time base setting of 100 μs/div. This results in:

$$B = \frac{8 \ \text{div}}{25.5 \ (\text{div})^2 \times \dfrac{100 \times 10^{-6} \ \text{sec}}{\text{div}}} \simeq 3.1 \ \text{kHz}$$

The Peak-Average Method

$$\left(B = \frac{V_p}{V_{av}} \ fp \right)$$

Theory—Geselowitz[2] and others show that the effective impulse bandwidth is related to the peak and average of the output pulse envelope as follows,
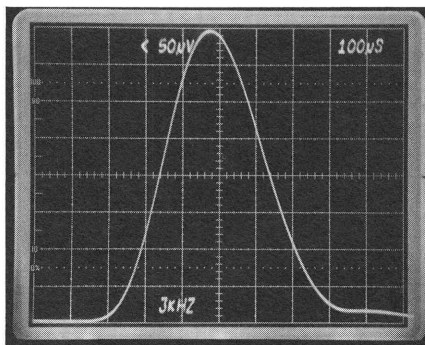
$$B = \frac{V_p}{V_{av}} \ fp$$

where B is the impulse bandwidth, V_p is the peak of the output envelope response, V_{av} is the average of the output envelope response, and f_p is the input pulse rate. The only requirement is that the input pulses be narrow enough to behave as impulses, and that the pulse rate (f_p) be less than approximately one-fifth of the bandwidth so there is no overlap between pulses. Thus, the instrument responds as to a single pulse.

Measurement—This technique is particularly applicable to the 7L5 because the instrument has both peak and average display functions.

The input waveform was a train of narrow pulses at a 500 Hz rate. Spectrum Analyzer gain was set for an output peak response of 8 divisions. The output average response was 1.3 divisions. Hence:

$$B = \frac{8 \ \text{divs}}{1.3 \ \text{divs}} \times 0.5 \ \text{kHz} \simeq 3.1 \ \text{kHz}$$

The Calibrated Impulse Method

$$\left(B = \frac{V_{psw}}{S.I.} \right)$$

Theory—This is an obvious technique and uses a train of pulses of known spectral intensity. One may use a commercially available calibrated impulse generator; a pulse train the spectral intensity of which is measured using a circuit of known impulse bandwidth; or a pulse train the spectral intensity of which is calculated from a knowledge of the pulse voltage and time parameters.

Figure 1—*Voltage response of 3-kHz resolution bandwidth position.*

Figure 2—*Impulse response of 3-kHz resolution bandwidth position.*

A sine wave generator is substituted for the impulse generator. The output level of the sine wave generator is adjusted to the same peak output response as that of the impulse generator. *The impulse bandwidth is then the ratio of the peak sine wave input level (V_{psw}) to impulse spectral intensity (S.I.).*

Measurement—The input signal was a train of pulses 0.8 V in amplitude and 0.15 μs in duration. The impulse spectral intensity equals twice the area of the pulse. Hence:

$$\text{S.I.} = 2 \times 0.8 \text{ V} \times 0.15 \times 10^{-6} \text{s}$$
$$= 0.24 \times 10^{-6} \text{ V-s}$$

This corresponds to 240 μV/kHz. A sinewave signal source was then substituted. The level was adjusted to give the same peak output response as the impulses. The sinewave generator output was 490 μV rms, which corresponds to 693 μV peak. Hence:

$$B = \frac{693 \ \mu V}{240 \ \mu V/kHz} \simeq 2.9 \text{ kHz}$$

A variation on this technique, developed by the National Bureau of Standards[3], is particularly useful at higher frequencies. The method involves a train of pulsed-rf carrier as the calibrated spectral intensity standard.

Computed Impulse Bandwidth
($B \simeq B_6$)

Theory—Sabaroff has shown[4,5] that, for a cascade of synchronously tuned circuits, the impulse bandwidth relates to the 3 dB and 6 dB bandwidths as follows:

$$\frac{B}{B_3} = \frac{2 \ \pi}{4 \sqrt{\pi} \ I_n \ \sqrt{2}} = 1.5$$

$$\frac{B}{B_6} = \frac{2 \ \pi}{4 \sqrt{\pi} \ I_n \ 2} = 1.06$$

Measurement—From Figure 1, we note that the 6-dB bandwidth is 2.95 kHz. Hence:

$$B = 2.95 \text{ kHz} \times 1.06 \simeq 3.1 \text{ kHz}$$

Impulse Bandwidth of 7L5
The agreement between results shows that all of the methods are accurate. The choice of best technique depends on the if filter construction of the spectrum analyzer in question; e.g. single-stage or multi-stage, single-tuned or double-tuned, etc. For the 7L5, we recommend the following:

1. Use the resolution bandwidth as a convenient estimate of impulse bandwidth[6].

2. Measure the 6-dB-down bandwidth to eliminate the possible ±20% inaccuracy in the resolution bandwidth specification.

3. Those looking for a very high confidence level might use two methods such as 1.06 B_6 and peak-average measurement as cross-checks*.
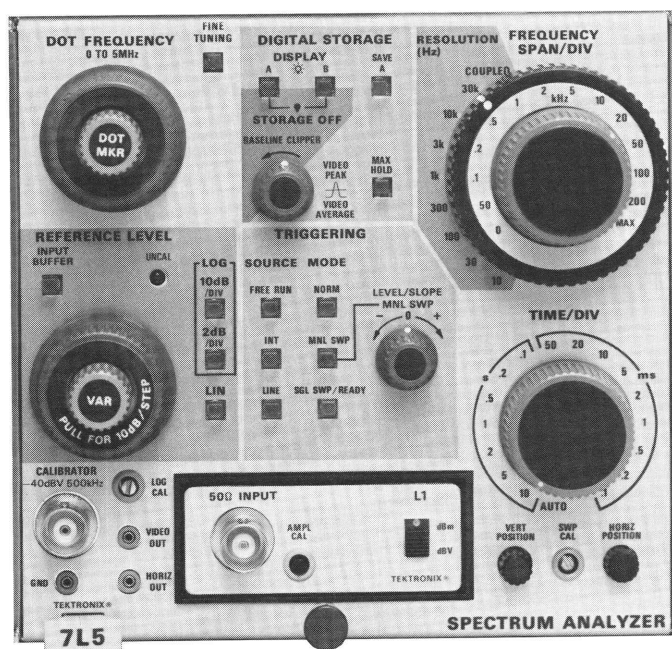
[4]Sabaroff—"Impulse Spectrum Analysis for Calibration of Impulse Noise Generators"—First Conf. on RFI Reduction, 1954, Conducted by IIT.

[5]Sabaroff—"Impulse Excitation of a Cascade of Series Tuned Circuits" —Proc. IRE Dec. 1944.

[6]Resolution bandwidth for Tektronix spectrum analyzers is specified at

[3]"Broadband Pulsed/CW Calibration Signal Standard for Field Intensity Receivers", NBSIR 74-371.

the 6dB down points. This and other aspects of resolution is discussed in Engelson—"Understand Resolution for Better Spectrum Analysis"—Microwaves. December 1974.

*The 10-kHz and 30-kHz resolution bandwidth position on the 7L5 are not based on single tuned amplifiers. The impulse bandwidth here is closer to 0.75 X B_6 than 1.06 X B_6.

III. Practical Pulses

Since mathematical impulses are not physically realizable, it follows that real pulses cannot appear as impulses under all conditions. Let us look at this problem.

The Fourier spectral distribution of a rectangular pulse of amplitude A and time duration t_0 is

$$F\ (f) = 2\ At_0\ \frac{\sin\ (\pi ft_0)}{\pi ft_0}.$$

(See reference 1)

The constant term ($2At_0$) is twice the area of the pulse, and is the same as the impulse spectral intensity. So long as $\sin\ (\pi ft_0)/\pi ft_0$ is close to unity, we may pretend that the pulse is an impulse. As we observe the spectrum at higher frequencies, however, we may no longer do so. *A good rule of thumb for the frequency limit where a pulse may be considered as an impulse is a pulse width (t_0) frequency product ($_f$) equal to 0.1. The expression*

$$\frac{\sin\ (0.1\pi)}{0.1\pi}\ = 0.98$$

for only 2% error.

The spectral shape at higher frequencies depends on the pulse shape. This shape may be computed by a Fourier analysis of the basic pulse as discussed in any reference on Fourier analysis. Figure 3 shows the spectrum of a rectangular pulse; Figure 4 shows the spectral distribution of a triangular pulse.
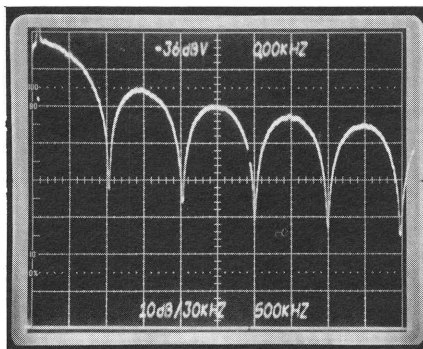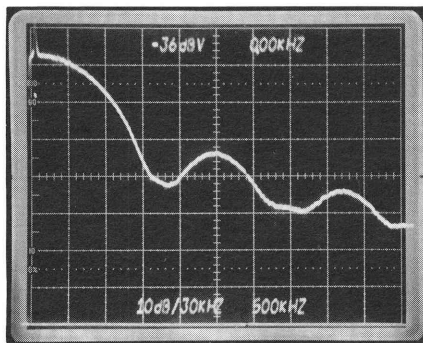

Figure 3—Sin x/x spectrum of rectangular pulse.

Figure 4—Spectrum of triangular pulse.

IV. Dynamic Range Calculation

Sensitivity—Pulse spectral intensity is given in units of volts/Hz. Thus, the wider the resolution bandwidth setting, the greater the output display level. A ten times increase in bandwidth means a ten fold increase in voltage output or a 100 fold (20 dB) increase in power. Internal spectrum analyzer noise, on the other hand, is random, increasing on a power per unit bandwidth basis. A ten times increase in resolution bandwidth means a ten fold (10 dB) increase in internal noise power. Hence, the input signal response increases faster than the internal noise as the resolution bandwith increases. In other words, unlike cw signals, *best pulse signal sensitivity is obtained at the widest resolution bandwidth settings.*

Resolution Settings — As indicated previously, the pulse width bandwidth product must be $t_0B \leq 0.1$ for the system to respond as though it were responding to an impulse. This puts an upper limit on the resolution bandwidth setting and the sensitivity of the system. In fact, for a rectangular pulse of amplitude V where the low frequency impulse spectral intensity is $2\ Vt_0$, the output level is 0.2 V at $t_0\ B = 0.1$. This is the maximum output level indication that can be obtained.

There is also a lower limit to the permissible bandwidth setting. This is established by the requirement that the pulse rate be less than the bandwidth as discussed in the section on the peak-average method of impulse bandwidth measurement.

Dynamic Range—Most spectrum analyzer front ends become non-linear at input levels above −10 dBm (+97 dBμV into 50 Ω). Thus *the signal to the input mixer should not exceed 97 dB above a microvolt.* Consider a worst case condition with the impulse spectrum extending over the full input range of the spectrum analyzer. For the 5 MHz 7L5 operating at 30 kHz resolution bandwidth the ratio of front end voltage to display voltage is

$$\frac{5\ MHz}{30\ kHz} = 167\ or\ 44\ dB.$$

For a maximum display level of
97-44 = +53 dBμV.

Instrument sensitivity at 30-kHz resolution bandwidth is −105 dBm, or +2 dBμV, producing a dynamic range of 51 dB. At narrower resolution bandwidth settings, the situation becomes worse. *At very wideband impulse noise distributions and narrow resolution bandwidth settings, it becomes impossible to make the measurement.*

V. Using Digital Storage

The 7L5 Spectrum Analyzer uses digital storage. This provides several features that are particularly useful in pulse signal measurements. First, let us consider what digital storage does.

Sampling—The display waveform is sampled at the rate of one sample every 9 μs. Thus, for example, at a sweep time setting of 10 ms/div, 1,111 samples are taken for every horizontal division. Not all of these samples are displayed. The screen is divided into 512 horizontal locations (256 for each spectral display when using split memory) with one display point per location. The samples within each location are compared and the largest value sample is displayed as the output when the instrument is in the video peak display mode. Again using 10 ms/div as an example, we note that about 22 samples are taken per each horizontal location, but only one of these is displayed. The displayed points are smoothed through an integrating filter to form a smooth continuous trace on the crt.

In addition to determining the largest, or peak, sample taken at each location, the system also computes the average of all the samples taken for each location. The user may choose to display the average rather than the peak by setting a front panel control.

Finally, the instrument has a maximum hold function wherein the new incoming peak levels are compared to those in memory, with the larger of these stored and displayed sequentially at each location.

The system has advantages and limitations. Let us consider the limitations first.

Limitations—When one of the 512 display locations shows a high level signal while the adjacent locations show no signal, it is necessary for the integrator to smooth a very short pulse that is only one display location wide. This can create a small amplitude error because of display rise time limitations. This difficulty is not likely to occur while viewing broadband noise or pulse spectra. It does occur while displaying discrete cw type signals. To avoid the problem, *the resolution bandwidth should not be less than* two display locations in width. This is equivalent to *about one twentieth of a horizontal division.*

The spectrum analyzer response to an impulse input is a narrow pulse such as that shown in Figure 2. Remembering that there are approximately 50 display locations per division, a minimum of 50 input pulses/div is required to get a value for each display location. Figure 5 shows a pulse spectrum in which the pulse rate compared to the spectrum analyzer sweep rate generated 50 pulses/div. The spectrum shape is clearly discernible, but there are many drop-outs where samples are missing. This can be compared to the less than 10 pulses per division that provide a satisfactory display in the "digital storage off" mode of Figure 6.
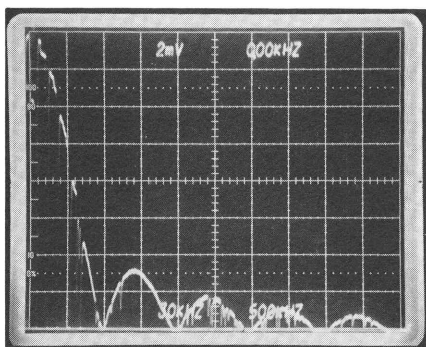
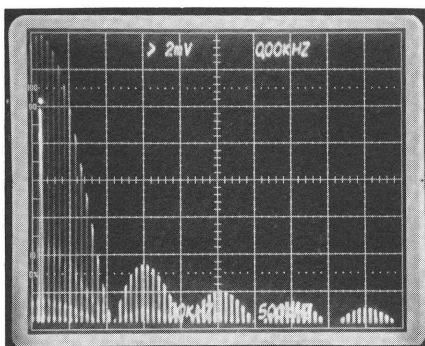*Figure 5—Pulse spectrum with digital storage "on." Pulse rate is 50 pulses/div.*

Figure 6—Pulse spectrum with digital storage "off." Pulse rate is 7 pulses/div.

Advantages—One advantage not available with conventional spectrum analyzers is the accurate measurement of average values. Among other things, this provides an easy way of measuring impulse bandwidth as discussed previously.

A smooth, integrated, display provides a more accurate determination of spectrum null points as demonstrated in Figure 3.

The maximum hold capability permits determination of spectrum shapes even at very low pulse rates.

Remember that this function displays the largest level occurring over a period of time. Figure 7 shows the display after 50 sweeps wherein the input pulse rate is only one pulse per division. Figure 8 shows the improvement after 200 spectrum analyzer sweeps. With the 7L5 capable of sweeping at 10 s/div, this is a very powerful technique for capturing random transient phenomena.

Finally, the digital storage can be turned off for those applications, such as pulse counting, that are best done by direct display.
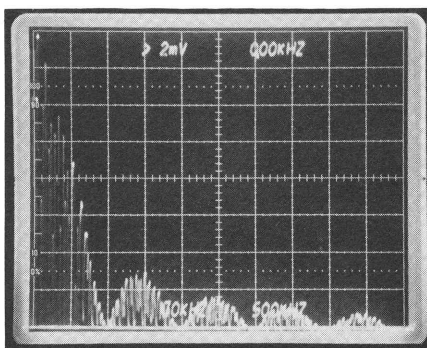

Figure 7—Pulse spectrum with digital storage "on." Pulse rate is one pulse/div. Using MAX HOLD function for 50 sweeps.
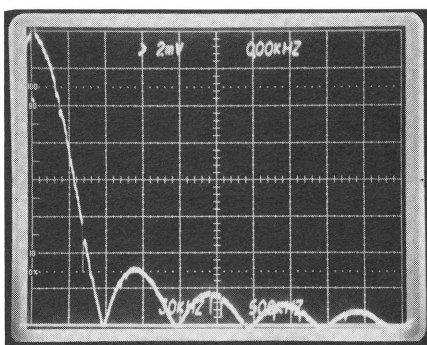

Figure 8—Same as Figure 7 after 200 sweeps.

VI. Summary

When making narrow pulse measurements using the 7L5 Spectrum Analyzer, the following points should be kept in mind.

1. Impulse bandwidth is very nearly equal to 6-dB resolution bandwidth.

2. The low frequency spectral intensity of a pulse is numerically equal to twice the pulse area. This approximation holds for $t_0 f < 0.1$.

3. For the spectrum analyzer to respond as to an impulse, it is necessary that: $t_0 B \leq 0.1$, $B >$ PRF.

4. For best sensitivity use the widest resolution bandwidth setting that is compatible with number 3 above.

5. For linear operation, do not exceed -10 dBm, ($+97$ dBμV in 50 Ω) into the front end mixer.

6. Sweep time/div should provide at least 5 input pulses/div for proper spectrum definition when digital storage is off.

7. Sweep time/div must provide a minimum of 50 input pulses/div for proper spectrum definition when digital storage is on.

8. Use the MAX HOLD function to capture the spectrum shape of slow rep rate or randomly occurring pulses.

9. Except for special application where the average value of a pulse train is desired, all pulse signal spectral displays should be in the peak vertical mode.