Tektronix

Technical Brief

Z-Profile™ Algorithm

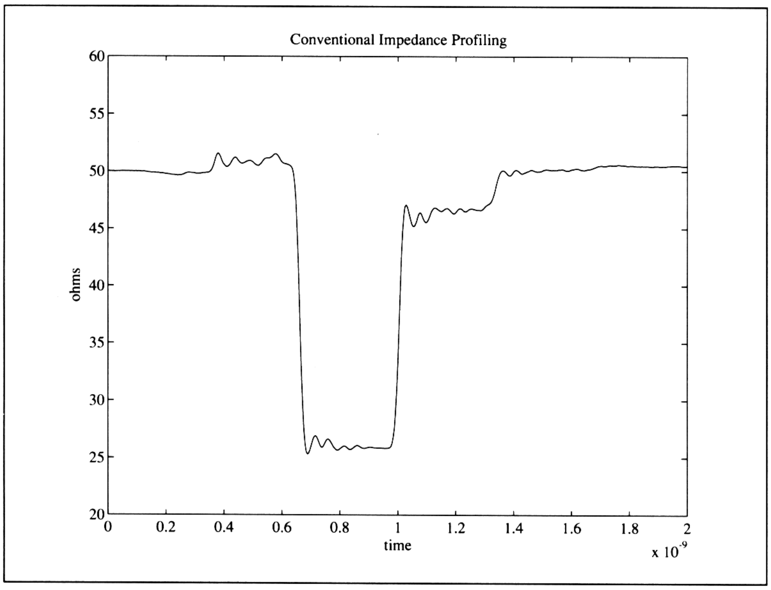

Figure 1. Calculated impedance profile for 5 cm, 25 ohm airline, using conventional method of determining impedance.

Conventional Method of Determining Impedance Profile

With the use of an SD-24 sampling head, the 11800 or CSA oscilloscopes can be used for Time Domain Reflectometry (TDR). TDR consists of sending out a voltage step and measuring the reflections which come back from this step (much like radar is used with aircraft). With the 11800/CSA oscilloscopes, TDR can be easily enabled through a TDR preset menu item (in the Sampling Head Fnc's menu) which will set up the step stimulus and set the screen to display the reflected waveform. Instead of displaying the voltage of the reflected waveform, the display shows the reflection coefficient, ρ. The reflection coefficient is calculated as,

$$\rho = \frac{V_{refl}}{V_{inc}}$$

where V_{refl} is the measured reflected voltage and V_{inc} is the incident step voltage amplitude (250 mV default). If cursors are brought up on screen, the cursor display will list a value of impedance, which is calculated from the formula,

$$Z = Z_{ref}\frac{1+\rho}{1-\rho}$$

where Z_{ref} is the reference value of impedance for the oscilloscope, 50 ohms.

The above formula has adequate accuracy for many situations; however, there are assumptions and approximations that can lead to incorrect results. The formula breaks down in two key situations: when the transit time in the device is small and the imperfections in the step become apparent, or when the DUT (device under test) has several impedance discontinuities that cause multiple reflections.

The errors associated with the conventional method of determining impedance can be easily demonstrated by looking at a real measurement of a 25 ohm, 5 cm (166 pS), airline standard. A TDR measurement was made on this standard and the resulting measured ρ was transformed, using the previous equation, to give the impedance profile shown in Figure 1. There are two errors associated with the impedance profile that can be traced to the non-ideality of the TDR step stimulus. A close-up of the actual TDR stimulus is shown in Figure 2. This stimulus differs from an ideal step because it has a finite risetime (about 31 pS for this step) and the voltage 'rings' at the top of the step. The source ringing is probably the most apparent error in the impedance profile of Figure 1. In the 25 ohm portion of the airline, the ringing causes an uncertainty of a few ohms in the impedance value. The finite risetime of the step is apparent in Figure 1 because the transition time from the 50 ohm to the 25 ohm level is always limited by the step risetime. With this particular device, the transition looks fairly abrupt. But for shorter devices the finite risetime of the step can significantly spread out the impedance profile.

Even with a perfect stimulus, using the conventional method, there would still be errors in the extracted impedance profile due to multiple reflections within the DUT. For the example DUT, the 25 ohm airline forms a resonant cavity between the 50 ohm sections. As the TDR step stimulus enters the 25 ohm section it undergoes several bounces within the cavity. The multiple reflections result in a delayed staircase structure found just after the 25 Ω section (beyond 1 nS in Figure 1). For this particular example, a connector and 50 ohm coax are physically beyond the 25 ohm airline, and yet the 'measured'

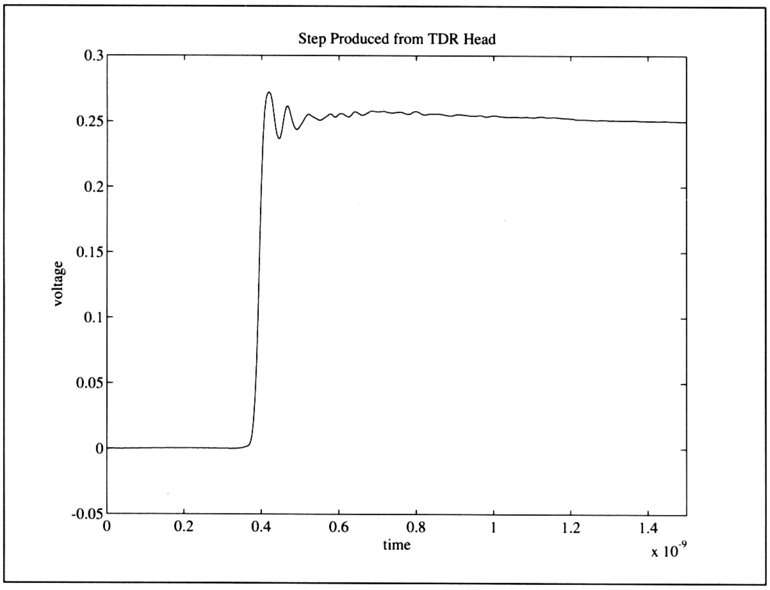

Figure 2. Typical step stimulus produced from SD-24 TDR head.

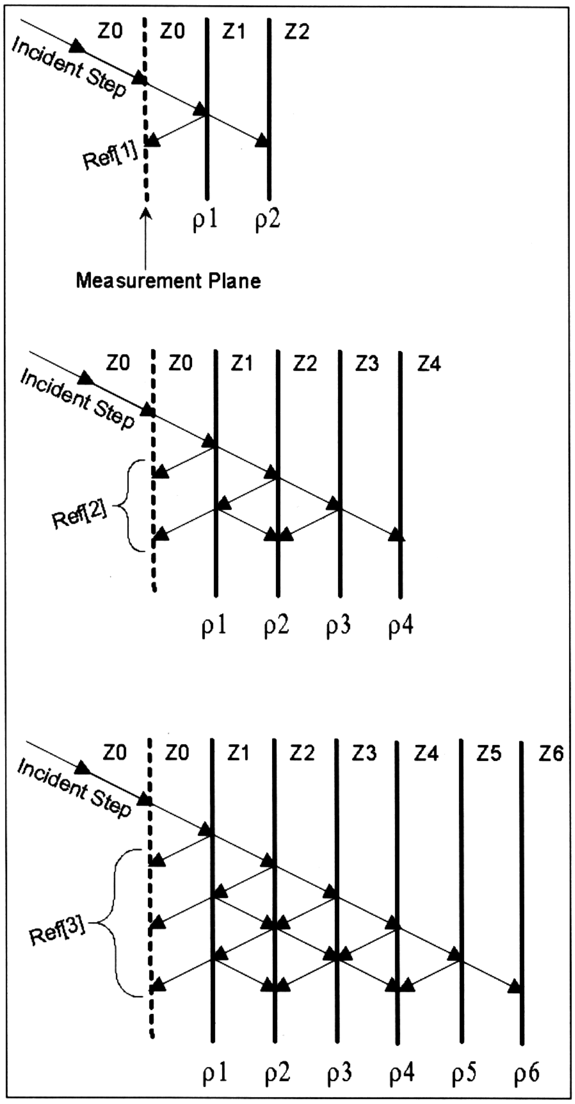

Figure 3. Snapshot of the incident and reflected waveforms at three different times.

value of impedance is about 46 ohms just past the device. The 25 ohm airline is a clear example of multiple reflections; however in general any time there are impedance discontinuities, reflections can occur and the user may not be aware that these reflections are distorting their results.

Z-Profile Algorithm for Improved Measurement of Impedance Profiles

The conventional method of measuring the device impedance suffers both from a sensitivity to the exact shape of the step excitation and from the assumption that there are not multiple reflections. The general problem of inverse scattering – the identification of layered wave propagation medium from a set of scattering data – is not unique to oscilloscope users and has, fortunately, been studied extensively by geophysicists. Much like oscilloscope users, geophysicists desire to determine the impedance layer structure. For geophysicists, the impedance layers are in the earth and the excitation is not a step, but an impulsive explosion at the surface. The technique which they have developed is called the direct dynamic deconvolution or layer peeling algorithm. This technique can be readily applied to TDR waveforms [1]-[3].

The first step in determining the impedance profile is to acquire a reflected TDR waveform and a second waveform which is representative of the incident step. Both waveforms must have the same time span and the same number of points. The incident step waveform is usually acquired by removing the device under test, placing a high quality short circuit at the end of the test fixture, and measuring the reflected waveform. It is important to use a high quality short because this data will be used to correct for imperfections in the source stimulus. Be sure to use a torque wrench to assure proper mating of the connectors.

Figure 3 shows three snapshots of a TDR signal incident on many layers of possibly different impedances. The incident step enters the structure and undergoes multiple reflections before being fully transmitted or reflected. At each interface in the structure, the reflection coefficient is given by

$$\rho_i = \frac{Z_i - Z_{i-1}}{Z_i + Z_{i-1}}$$

The Z-Profile algorithm will output an array of impedance layer values. The time spacing between the impedance layers will be one half of the time between sample points in the original data. For the Tektronix 11800 family of oscilloscope, the time interval between sample points can be less than 100 femto seconds, so the impedance partitioning in Z-Profile can be extremely fine.

The Z-Profile algorithm works with the sampled values of the incident and reflected waveforms. Conceptually, it is easier to understand the lattice diagrams of Figure 3, if the incident and reflected waveforms are thought of as a sum of rectangular pulses as shown in Figure 4. If the oscilloscope acquired the incident and reflected waveforms with 512 points, then each of these waveforms can conceptually be thought of as 512 independent rightward or leftward traveling rectangular pulses.

Looking again at Fig 3, with the spacing between layers chosen at one half of the sampling interval, each pulse can traverse two arrow segments between sampling inter-

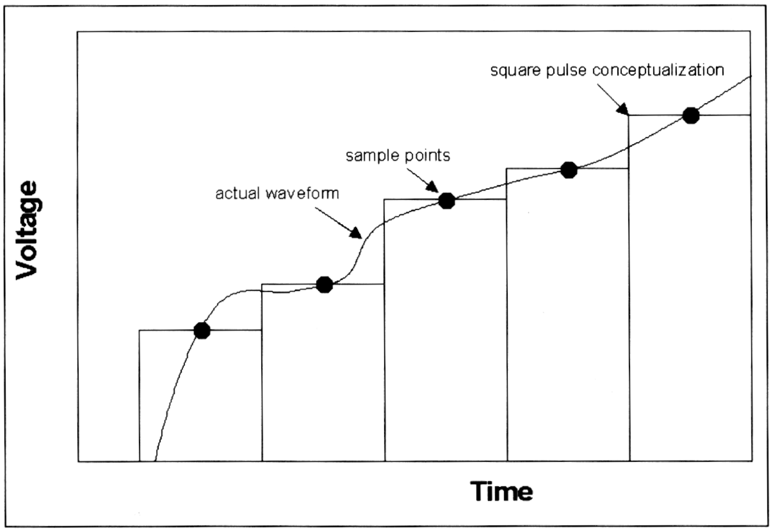

Figure 4. Conceptualization of measured waveforms. Solid line indicates the true voltage waveform, dots represent the sample points from the oscilloscope. Conceptually, it is useful to think of the waveform as made up of a series of independent rectangular pulses.

vals. Let *Ref[i]* and *Inc[i]* be the i'th component, or pulse rectangle, of the measured reflected waveform and incident waveforms respectively. Time i=1 is shown in the first snapshot of Figure 3. The first pulse on the incident wave has penetrated the impedance structure and part of the wave has bounced back off the first interface to be measured at the reference plane. The measured reflection coefficient can be written as:

$$Ref[1] = \rho_1 Inc[1]$$

In the second time interval, shown in the middle snapshot, the second incident pulse will reflect off of the first interface and in addition, part of the first pulse which was previously transmitted through the first interface will bounce off the second interface and find it's way to the reference plane. At i=2 the measured reflected waveform will be:

$$Ref[2] = t_1^2 \rho_2 Inc[1] + \rho_1 Inc[2]$$

where the t's in the above equations are the interface transmission coefficients. In later time intervals the situation gets more complicated as more and more incident pulses enter the structure and bounce among the impedance layers. The situation for i=3 is shown in the bottom snapshot of Figure 3. The measured reflected waveform can be written as

$$\mathrm{Re}\,f[3] = (t_1^2 t_2^2 \rho_3 - t_1^2 \rho_2^2 \rho_1) Inc[1] + t_1^2 \rho_2 Inc[2] + \rho_1 Inc[3]$$

One can continue to write out the reflected waveform in terms of the incident waveform and the media. For 512 points in the incident and reflected waveform, there will be 512 equations, which can be written more compactly in matrix form

$$\begin{pmatrix} \mathrm{Re}\,f[1] \\ \mathrm{Re}\,f[2] \\ \mathrm{Re}\,f[3] \\ \vdots \\ \mathrm{Re}\,f[512] \end{pmatrix} = \begin{pmatrix} c_1 & 0 & 0 & \cdots & 0 \\ c_2 & c_1 & 0 & & 0 \\ c_3 & c_2 & c_1 & & \vdots \\ \vdots & & & \ddots & 0 \\ c_{512} & c_{511} & \cdots & c_2 & c_1 \end{pmatrix} \begin{pmatrix} Inc[1] \\ Inc[2] \\ Inc[3] \\ \vdots \\ Inc[512] \end{pmatrix}$$

The c's in the matrix are the coefficients of the equations, for example $c_1=\rho_1$. The triangular matrix can be solved fairly easily. The first values of the incident and reflected waveforms, *Ref[1]* and *Inc[1]*, are used to solve for c_1. Once c_1 has been determined the next equation can be used to solve for c_2 and this process can be repeated until all the c's have been calculated. Once the values of all the c's are known, the various reflection coefficients, and thus the impedances can be calculated to produce an impedance profile for the device under test. The details of solving the reflection coefficients from the c's are more fully discussed in the references [1]-[3].

The solution for the matrix is closely related to performing a deconvolution of the reflected signal from the incident signal. However, the Z-Profile algorithm does more than just a deconvolution. If the reflected waveform were only deconvolved from the incident waveform, the ringing in the source and the finite risetime would be corrected; however, there would not be any corrections for the multiple reflections in the device. The Z-Profile algorithm provides the improvements of a deconvolution and also corrects for the multiple reflections.

In theory, the impedance profile that is extracted is an exact solution. The previous problems with finite source risetime, ringing in the source step and multiple reflections have all been taken into account in the Z-Profile algorithm. In practice, when this method is applied to noiseless data, it does extract the exact impedance profile. However, for real data with noise, the standard Z-Profile algorithm must be modified slightly.

Reducing the Effect of Noise

Without the presence of noise, the Z-Profile algorithm will correctly extract the impedance profile for the DUT. However, noise in the data will introduce errors and these errors can propagate and produce larger errors for the impedance values further in the device. This can be seen from the matrix equations presented in the last section. If there is noise on the data, c_1 may be determined incorrectly, and the error in c_1 will then be propagated and added as an error for c_2, c_3, c_4, etc. In general, any error in early c values will be propagated and add additional error to later c values [4]. Figure 5 shows the impedance profile that is produced if the z profile algorithm is applied to the real, noisy data of Figs 1-2. Initially the algorithm extracts the correct value of impedance; however, errors rapidly develop and the solution 'blows up' after about 200 pS.

In the Z-Profile algorithm, the initial c_1 value is determined from the first point in the incident step and reflected waveforms. Typically, the magnitude of these first data points is very small, less than 1 mV, and the noise in the system itself may be only slightly less than 1 mV. With the noise almost as large as the signal, there is the potential for tremendous error in the determination of the initial c_1 value in the matrix. This error will propagate to the rest of the coefficients. To reduce the sensitivity to noise, the incident step is preprocessed and clipped at some value of its total height (typically 15-30%). Figure 6 shows what the incident step waveform looks like when it has been clipped at a 20% level. This clipped waveform is then used in the Z-Profile algorithm to lessen the effect of noise. The clipping % is, in effect, setting a noise threshold.

Figure 7 shows the conventional and Z-Profile impedance distribution that is obtained when the clipped step of Figure 6 is applied to the measured TDR waveform for a 25 ohm airline. The Z-Profile algorithm has removed the effects of multiple reflections so that the impedance returns to 50 ohms beyond the device. In addition, the ringing which was seen in the 25 ohm section has been significantly reduced. Figure 8 shows the conventional and Z-Profile method impedance profiles for a TO-220 IC package [5]. In the conventional extraction, the risetime of the step has been slowed down and there are enough multiple reflections that it is difficult to see the location of the short circuit at the end of this package. However, after the Z-Profile algorithm has been applied the end of the package is much clearer and the inductances in the package are more defined.

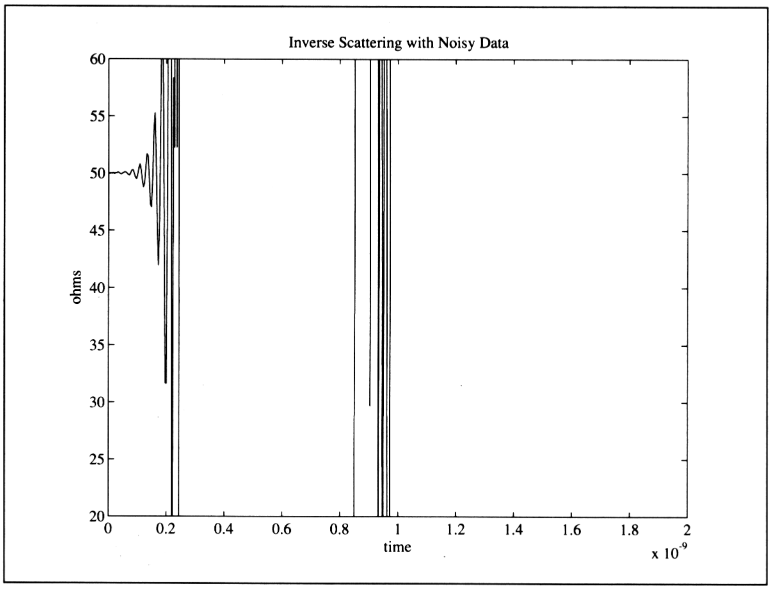

Figure 5. Impedance profile which is extracted with Z-Profile algorithm when applied to noisy data.

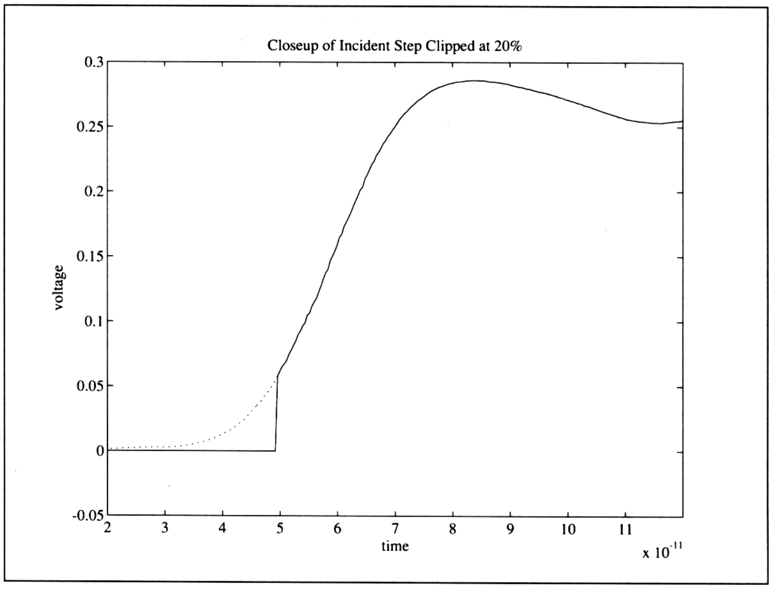

Figure 6. Incident step which has been processed to set a noise threshold. This step was clipped at 20% of its final value. The original step is shown as a dotted line.

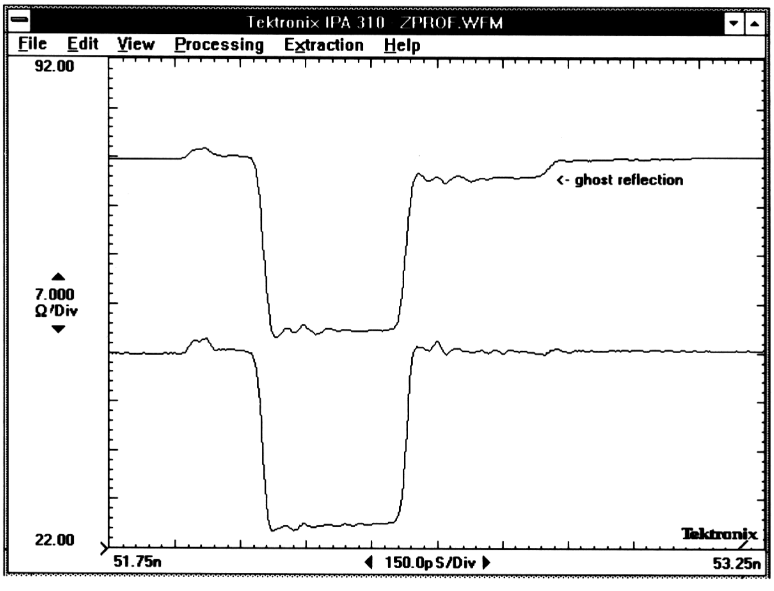

Figure 7. Impedance profile obtained from conventional method, top, and from the Z-Profile algorithm (bottom) for a 25 ohm, 5 cm long airline.

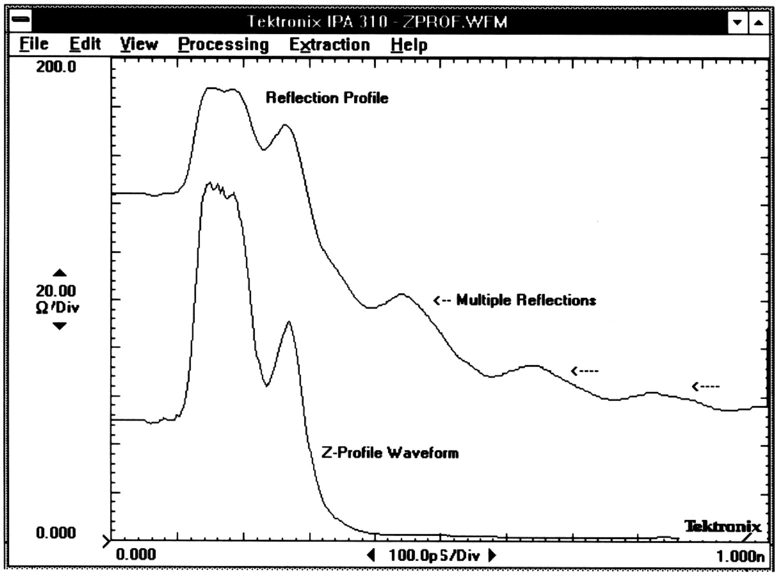

Figure 8. Impedance profile obtained from conventional method, top, and from the Z-Profile algorithm (bottom) for TO-220 IC package. Note the clearer definition in the Z-Profile waveform.

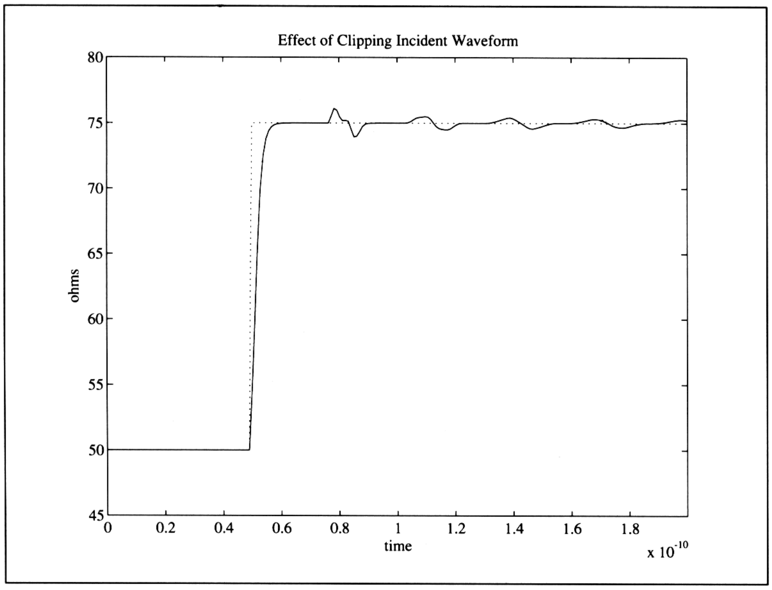

Figure 9. Artifacts introduced in impedance profile from clipping incident step. The actual profile is shown as a dotted line and the Z-Profile distribution is shown as a solid line.

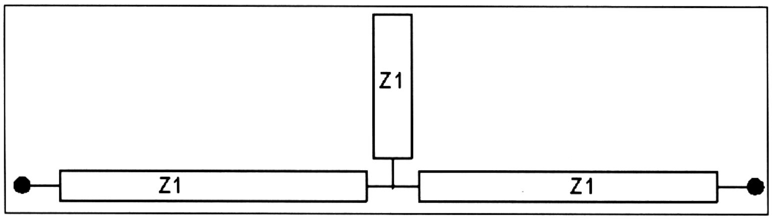

Figure 10. Relatively simple branched transmission line structure which will not be correctly extracted by Z-Profile algorithm.

Limitations of Z-Profile Algorithm

Clipping the incident step waveform reduces the sensitivity that the Z-Profile algorithm has to noise. Unfortunately this preprocessing can also produce errors in the extracted impedance profile. When the incident waveform is clipped, data is deleted which is not correspondingly deleted from the reflected signal. Fortunately, in most cases, the artifacts introduced by clipping are relatively minor. Figure 9 shows the actual and Z-Profile impedance profiles that were obtained for a 50 ohm to 75 ohm transition. Two different artifacts are evident from the Z-Profile algorithm. The first artifact is exponential convergence from 50 to 75 ohms. Typically it takes about one half of the step rise time for the solution to settle in at a final value. The second artifact is the repeated impedance ripple further into the device. Typically this ripple appears at a distance of several risetimes further into the device.

Because of the exponential convergence, it can be difficult to discern structures that extend in time significantly less than the source risetime. The Z-Profile algorithm does provide some effective risetime improvement, but clipping places a lower limit on the improvement of about 1/3 of the incident signal risetime at the device. If, for example, your source has a risetime of 35 pS then you should not expect to discern structures which are less than 12 pS long. It should also be noted, that it is not the risetime out of the source which is important, but instead the risetime of the incident step at the device. The step from the source may have a risetime of only 30 pS, but if you do not provide a high frequency connection to your device then the device risetime may be significantly greater. Suppose, for example, you are testing an IC package. The lead of this package may have a substantial inductance. If this is the case the risetime could be slowed to 200 pS within the package. With a 200 pS risetime step, the Z-Profile algorithm will only be able to discern structures 65 pS or longer inside the package.

In addition to the limitations caused by clipping the incident waveform there is also a fundamental restraint in the Z-Profile algorithm in that it only works for series structures. The Z-Profile algorithm will give incorrect results for branched structures. Figure 10 shows a branched structure with which the algorithm will break down. When an incident step signal is applied to the branched structure in Figure 10, part of the signal will divert down the open stub, reflect in the stub a few times and eventually this power will head back out to the source. The reflected TDR signal will then have a ripple component corresponding to the signal that bounces in the open stub. The Z-Profile algorithm will attempt to represent this device by a series connection of different impedance sections. Clearly the Z-Profile algorithm will be unsuccessful in representing an electrically long branched structure as a series structure. The structure shown in Figure 10 is one type of branched structure. In general any time multiple paths are possible, the Z-Profile algorithm may extract incorrect values of impedance. One common case where multiple paths are possible is when there is coupling between adjacent lines. If the coupling between adjacent lines is more than ten percent, then the Z-Profile algorithm should be used with caution. In general it is always advisable to verify your models by comparing the measured TDR waveforms with those predicted from a model.

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